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The gravitational mechanism to generate mass

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Abstract
The purpose of this work is to show that the gravitational interaction is able to generate mass for all bodies. The condition for this is the existence of an energy distribution represented by the vacuum or the cosmological constant.

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1. Introduction

Although the theory of general relativity may be understood as completely independent from the Machian idea that inertia of a body $\mathbb{A}$ is related to the global distribution of energy of all particles existing in the Universe, we must recognize its historical value in making the ideology that enabled Einstein to start his journey toward the construction of a theory of gravitation [1]. During the 20th century, the idea of associating the dependence of local characteristics of matter with the global state of the universe came up now and then but without producing any reliable mechanism that could support such a proposal [2]. Even the concept of mass—that pervades all gravitational processes—did not find a realization of such dependence on the global structure of the universe. In contrast, the most efficient mechanism and the one that has performed an important role in the field of microphysics came from elsewhere, namely the attempt to unify forces of a non-gravitational character, such as long-range electrodynamics with decaying phenomena described by weak interaction. Indeed, the Higgs model produced an efficient scenario for generating mass to the vector bosons [3] that goes in the opposite direction of the proposal of Mach. This mechanism starts with the transformation of a global symmetry into a local one and the corresponding presence of vector gauge fields. Then, a particular form of the dynamics represented by $L_{\text{int}}(\varphi)$ of self-interaction of an associated scalar field in its fundamental state represented by an energy–momentum tensor given by $T_{\mu\nu} = L_{\text{int}}(\varphi_0) g_{\mu\nu}$ appears as the vehicle which provides mass to the gauge fields.

In this paper, we shall describe a new mechanism for generation of mass that is a realization of Mach's idea. Our strategy is to couple the field (scalar, spinor, vector and tensor) non-minimally to gravity through the presence of terms involving explicitly the curvature of

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spacetime. The distribution of the vacuum energy of the rest-of-the-universe (ROTU) is represented by a cosmological term Λ. The effect of Λ by the intermediary of the dynamics of the metric of spacetime in the realm of general relativity is precisely to give mass to the field. Although this mass depends on the cosmological constant, its value cannot be obtained *a priori*[2].

1.1. Mass for scalar field: a trivial case

The Higgs mechanism appeals to the intervention of a scalar field that was proposed as the vehicle which provides mass to the gauge vector field W_μ. For the mass to be a real and constant value (a different value for each field), the scalar field φ must be in a minimum state of its potential V. This fundamental state of the self-interacting scalar field has an energy distribution given by $T_{\mu\nu} = V(\varphi_0) g_{\mu\nu}$. A particular form of self-interaction of the scalar field φ allows the existence of a constant value $V(\varphi_0)$ that is directly related to the mass of W_μ. This scalar field has its own mass, the origin of which rests unclear. This mechanism was generalized for other fields and it is adopted in the realm of high-energy physics as the way mass for all fields is obtained.

Although the concept of mass pervades most of all analyses involving gravitational interaction, it is an uncomfortable situation that still to date there has been no successful attempt to derive a mechanism by means of which mass is understood, a direct consequence of a dynamical process depending on gravity [4][3].

The main idea concerning inertia in the realm of gravity according to the origins of general relativity goes in the opposite direction of the mechanism in the territory of the high-energy physics. Indeed, while the Higgs mechanism explores the reduction of a global symmetry into a local one, the Mach principle suggests a cosmical dependence of local properties, making the origin of the mass of a given body to depend on the structure of the whole Universe. In this way, there ought to exist a mechanism by means of which this quantity—the mass—depends on the state of the universe. How to understand such broad concept of mass? Let us describe an example of such mechanism in order to see how this vague idea can achieve a quantitative scheme[4].

We start by considering the Mach principle as the statement according to which the inertial properties of a body $\mathbb{A}$ are determined by the energy–momentum throughout all space. How could we describe such universal state that takes into account the whole contribution of the ROTU onto $\mathbb{A}$? There is no simpler way than to consider this state as the most homogeneous one and relate it to what Einstein attributed to the cosmological constant or, in modern language, the vacuum of all remaining bodies. This means to describe the energy–momentum distribution of all complementary bodies of $\mathbb{A}$ as

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

[2] Let us remark that due to the method of obtaining mass using the gravity mechanism, the notion of totality (or ROTU) admits two alternative interpretations. This is related to the fact that when we deal with the vacuum represented by the distribution of energy by $T_{\mu\nu} = \lambda g_{\mu\nu}$, it is completely irrelevant—for the present gravitational mechanism of providing mass—if the parameter λ has a classical global origin (the Universe) identified with the cosmological constant introduced by Einstein, or a local quantum one (the environment) identified with the vacuum of quantum fields.

[3] In this reference one can find further references to previous attempts and in particular the beautiful proposal of Hoyle and Narlikar.

[4] At this point one should note that the expression ROTU may not have a strict cosmological meaning, but instead is a short term to represent the whole background of matter that really affects $\mathbb{A}$. However, this is not mandatory. The term ROTU concerns the environment of $\mathbb{A}$, that is the whole domain of influence on $\mathbb{A}$ of the remaining bodies in the Universe.

Let φ be a massless field, the dynamics of which is given by the Lagrangian

$$L_\varphi = \tfrac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi.$$

In the framework of general relativity, its gravitational interaction is given by the Lagrangian

$$L = \frac{1}{\kappa_0}R + \frac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi + B(\varphi)R - \frac{\Lambda}{\kappa_0} \tag{1}$$

where for the time being the dependence of B on the scalar field is not fixed. This dynamics represents a scalar field non-minimally coupled to gravity. The cosmological constant is added for the reasons presented above and represents the influence of the ROTU on φ. We shall see that Λ is the real responsible to provide mass for the scalar field. This means that if we set $\Lambda = 0$, the mass of the scalar field should vanish.

Independent variation of φ and $g_{\mu\nu}$ yields

$$\Box\varphi - RB' = 0 \tag{2}$$

$$\alpha_0\left(R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu}\right) = -T_{\mu\nu} \tag{3}$$

where we set $\alpha_0 \equiv 2/\kappa_0$ and $B' \equiv \partial B/\partial\varphi$. The energy–momentum tensor is given by

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi g_{\mu\nu} + 2B\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 2\nabla_\mu\nabla_\nu B - 2\Box Bg_{\mu\nu} + \frac{\Lambda}{\kappa_0}g_{\mu\nu}. \tag{4}$$

Taking the trace of equation (3), we obtain

$$(\alpha_0 + 2B)R = -\partial_\alpha\varphi\partial^\alpha\varphi - 6\Box B + \frac{4\Lambda}{\kappa_0}. \tag{5}$$

Inserting this result into equation (2) yields

$$\Box\varphi + \mathbb{Z} = 0 \tag{6}$$

where

$$\mathbb{Z} \equiv \frac{B'}{\alpha_0 + 2B}\left(\partial_\alpha\varphi\partial^\alpha\varphi + 6\Box B - \frac{4\Lambda}{\kappa_0}\right)$$

or, equivalently,

$$\mathbb{Z} = \frac{B'}{\alpha_0 + 2B}\left(\partial_\alpha\varphi\partial^\alpha\varphi(1 + 6B'') + 6B'\Box\varphi - \frac{4\Lambda}{\kappa_0}\right).$$

Therefore, the scalar field acquires an effective self-interaction through the non-minimal coupling with the gravitational field. At this stage, it is worth selecting among all possible candidates of B a particular one that makes the factor on the gradient of the field to disappear in the expression of $\mathbb{Z}$ by setting

$$B = a + b\varphi - \tfrac{1}{12}\varphi^2$$

where a and b are the arbitrary parameters. The quantity a makes only a renormalization of the constant $1/\kappa_0$ and the parameter b is responsible for distinguishing different masses for different fields. Making a translation on the field

$$\Phi = -\varphi + 6b,$$

it follows that

$$\Box\Phi + \mu^2\Phi = 0 \tag{7}$$

where

$$\mu^2 = \frac{2\Lambda}{3}\frac{\kappa_{\text{ren}}}{\kappa_0}. \tag{8}$$

where

$$\kappa_{\text{ren}} = \frac{1}{\alpha_0 + 2a + 6b^2}.$$

Thus, as a result of the above process, the scalar field acquires a mass μ that depends on Λ. If Λ vanishes, then the mass of the field vanishes. This is a net effect of the non-minimal coupling of gravity with the scalar field. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe in the state in which all existing matter is in the corresponding vacuum. The values of different masses for different fields are contemplated in the parameter b. Note however that this case contains the renormalization of the gravitational constant and conformal coupling of the scalar with the gravitational field. A more general procedure makes appeal not only to the non-minimal coupling with gravity, but also to the self-coupling interaction in a curved spacetime. Let us turn now to this case.

1.2. Mass for scalar field-II

Let us now analyze a more general scenario to provide mass to a scalar field. We start from the Lagrangian that describes a massless field φ that is

$$L_\varphi = \tfrac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi.$$

The gravitational interaction yields the modified Lagrangian

$$L = \frac{1}{\kappa}R + \frac{1}{2}W(\varphi)\partial_\alpha\varphi\partial^\alpha\varphi + B(\varphi)R - \frac{1}{\kappa}\Lambda \tag{9}$$

where for the time being the dependence of B and W on the scalar field is not fixed. We set $\hbar = c = 1$.

This dynamics represents a scalar field coupled non-minimally with gravity. There is no direct interaction between φ and the ROTU, except through the intermediary of gravity described by a cosmological constant Λ. Thus, Λ represents the whole influence of the ROTU on φ.

Independent variation of φ and $g_{\mu\nu}$ yields

$$W\,\Box\varphi + \tfrac{1}{2}W'\partial_\alpha\varphi\partial^\alpha\varphi - B'R = 0 \tag{10}$$

$$\alpha_0\left(R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu}\right) = -T_{\mu\nu} \tag{11}$$

where $\alpha_0 \equiv 2/\kappa$ and $B' \equiv \partial B/\partial\varphi$. The energy–momentum tensor defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L)}{\delta g^{\mu\nu}}$$

is given by

$$T_{\mu\nu} = W\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}W\partial_\alpha\varphi\partial^\alpha\varphi g_{\mu\nu} + 2B\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 2\nabla_\mu\nabla_\nu B - 2\,\Box\,Bg_{\mu\nu} + \frac{1}{\kappa}\Lambda g_{\mu\nu}. \tag{12}$$

Taking the trace of equation (11), we obtain

$$(\alpha_0 + 2B)R = -\partial_\alpha\varphi\partial^\alpha\varphi(W + 6B'') - 6B'\,\Box\varphi + 4\frac{\Lambda}{\kappa} \tag{13}$$

where we used that $\Box\,B = B'\,\Box\varphi + B''\partial_\alpha\varphi\partial^\alpha\varphi$.

Inserting this result back into equation (10) yields

$$\mathbb{M}\,\Box\varphi + \mathbb{N}\partial_\alpha\varphi\partial^\alpha\varphi - \mathbb{Q} = 0 \tag{14}$$

where

$$\mathbb{M} \equiv W + \frac{6(B')^2}{\alpha_0 + 2B}$$

$$\mathbb{N} \equiv \frac{1}{2}W' + \frac{B'(W + 6B'')}{\alpha_0 + 2B}$$

$$\mathbb{Q} = \frac{4\Lambda B'}{\kappa(\alpha_0 + 2B)}.$$

Thus, through the non-minimal coupling with the gravitational field, the scalar field acquires an effective self-interaction. At this point, it is worth selecting among all possible candidates of B and W the particular ones that make the factor on the gradient of the field to disappear on the equation of motion (EOM) by setting $\mathbb{N} = 0$. This condition will give W as a function of B:

$$W = \frac{2q - 6(B')^2}{\alpha_0 + 2B} \tag{15}$$

where q is a constant. Inserting this result into equation (14) yields

$$\Box\varphi - \frac{2\Lambda}{q\kappa}B' = 0. \tag{16}$$

At this point, one is led to set

$$B = -\frac{\beta}{4}\varphi^2$$

to obtain

$$\Box\varphi + \mu^2\varphi = 0 \tag{17}$$

where

$$\mu^2 \equiv \frac{\beta\Lambda}{q\kappa}. \tag{18}$$

For the function W, we obtain

$$W = \frac{2q - 3\beta^2\varphi^2}{2\alpha_0 - \beta\varphi^2}.$$

One should set $q = \alpha_0$ in order to obtain the standard dynamics in case β vanishes. Using units where $\hbar = 1 = c$, we write

$$\mathbb{L} = \frac{1}{\kappa}R + \frac{2q - 3\beta^2\varphi^2}{2(2\alpha_0 - \beta\varphi^2)}\partial_\alpha\varphi\partial^\alpha\varphi - \frac{1}{4}\beta\varphi^2 R - \frac{\Lambda}{\kappa}.$$

Thus, as a result of the gravitational interaction, the scalar field acquires a mass μ that depends on the constant β and on the existence of Λ:

$$\mu^2 = \frac{\beta\Lambda}{2}. \tag{19}$$

If Λ vanishes, then the mass of the field vanishes. The net effect of the non-minimal coupling of gravity with the scalar field corresponds to a specific self-interaction of the scalar field. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe—represented by the cosmological constant—in the state in which all existing matter is in the corresponding vacuum. The values of different masses for different fields are contemplated in the parameter β.

1.3. Renormalization of the mass

The effect of the ROTU on a massive scalar field can be analyzed through the same lines as above. Indeed, let us consider the case in which there is a potential $V(\varphi)$:

$$L = \frac{1}{\kappa}R + \frac{W}{2}\partial_\alpha\varphi\partial^\alpha\varphi + B(\varphi)R - V(\varphi) - \frac{\Lambda}{\kappa}. \tag{20}$$

The equation for the scalar field is given by

$$W\,\Box\varphi + \tfrac{1}{2}W'\partial_\alpha\varphi\partial^\alpha\varphi - B'R + V' = 0. \tag{21}$$

Use the equation for the metric to obtain the scalar of curvature in terms of the field and Λ. It then follows that terms in $\partial_\alpha\varphi\partial^\alpha\varphi$ are absent, if we set

$$W = \frac{2q - 6(B')^2}{\alpha_0 + 2B}$$

where q is a constant. For the case in which $B = -\beta\varphi^2/4$ and for the potential

$$V = \frac{\mu_0}{2}\varphi^2$$

and choosing $q = 2/\kappa$ (in order to obtain the standard equation of the scalar field in case $B = 0$) yields

$$\Box\varphi + \left(\mu_0^2 + \beta\Lambda/2\right)\varphi + \frac{\beta\mu_0^2}{4}\kappa\varphi^3 = 0. \tag{22}$$

This dynamics is equivalent to the case in which the scalar field shows an effective potential (in the absence of gravity) of the form

$$V_{\rm eff} = \left(\mu_0^2 + \beta\Lambda\right)\frac{\varphi^2}{2} + \frac{\beta\mu_0^2\kappa}{16}\varphi^4.$$

Thus, the net effect of the gravitational interaction for the dynamics driven by (20) is to renormalize the mass from the bare value μ_0 to the value

$$\mu^2 = \mu_0^2 + \frac{\beta\Lambda}{2}.$$

We can then contemplate the possibility that all bodies represented by a scalar field could have the same bare mass and as a consequence of gravitational interaction acquires a split into different values characterized by the different values of β. This result is not exclusive for the scalar field but is valid for any field.

2. The case of fermions

Let us now turn our attention to the case of fermions. The massless theory for a spinor field is given by the Dirac equation:

$$\mathrm{i}\gamma^\mu\partial_\mu\Psi = 0. \tag{23}$$

This equation is invariant under γ^5 transformation. In order to have mass for the fermion, this symmetry must be broken. Who is responsible for this?

2.1. Gravity breaks the symmetry

Electrodynamics appears in gauge theory as a mechanism that preserves a symmetry when one moves from a global transformation to a local one (spacetime-dependent map). Nothing is similar with gravity. In contrast, in the generation of mass through the mechanism that we are analyzing here, gravity is responsible to break the symmetry. In the framework of general relativity, the gravitational interaction of the fermion is driven by the Lagrangian

$$L = \frac{i\hbar c}{2}(\bar{\Psi}\gamma^{\mu}\nabla_{\mu}\Psi - \nabla_{\mu}\bar{\Psi}\gamma^{\mu}\Psi) + \frac{1}{\kappa}R + V(\Phi)R - \frac{1}{\kappa}\Lambda + L_{CT} \tag{24}$$

where the non-minimal coupling of the spinor field with gravity is contained in the term $V(\Phi)$ that depends on the scalar

$$\Phi \equiv \bar{\Psi}\Psi$$

which preserves the gauge invariance of the theory under the map $\Psi \rightarrow \exp(i\theta)\Psi$. Note that the dependence of Φ on the dynamics of Ψ breaks the chiral invariance of the massless fermion; a condition that is necessary for a mass to appear.

For the time being the dependence of V on Φ is not fixed. We have added a counter-term L_{CT} for reasons that will be clear later on. On the other hand, the form of the counter-term should be guessed, from the previous analysis that we made for the scalar case, that is, we set

$$L_{CT} = H(\Phi)\partial_{\mu}\Phi\partial^{\mu}\Phi. \tag{25}$$

This dynamics represents a massless spinor field coupled non-minimally with gravity. The cosmological constant represents the influence of the ROTU on Ψ.

Independent variation of Ψ and $g_{\mu\nu}$ yields

$$i\gamma^{\mu}\nabla_{\mu}\Psi + (RV' - H'\partial_{\mu}\Phi\partial^{\mu}\Phi - 2H\,\Box\,\Phi)\Psi = 0 \tag{26}$$

$$\alpha_0\left(R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu}\right) = -T_{\mu\nu} \tag{27}$$

where $V' \equiv \partial V/\partial\Phi$. The energy–momentum tensor is given by

$$T_{\mu\nu} = \frac{i}{4}\,\bar{\Psi}\gamma_{(\mu}\nabla_{\nu)}\Psi - \frac{i}{4}\nabla_{(\mu}\bar{\Psi}\gamma_{\nu)}\Psi + 2V\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 2\nabla_{\mu}\nabla_{\nu}V - 2\,\Box\,Vg_{\mu\nu}$$
$$+ 2H\partial_{\mu}\Phi\partial_{\nu}\Phi - H\partial_{\lambda}\Phi\partial^{\lambda}\Phi g_{\mu\nu} + \frac{\alpha_0}{2}\Lambda g_{\mu\nu}. \tag{28}$$

Taking the trace of equation (27), we obtain after some algebraic manipulation

$$(\alpha_0 + 2V + V')R = H'\,\Phi\partial_{\alpha}\Phi\partial^{\alpha}\Phi + 2H\Phi\,\Box\,\Phi - 6\,\Box\,V + 2\,\alpha_0\Lambda. \tag{29}$$

Inserting this result back into equation (26) yields

$$i\gamma^{\mu}\nabla_{\mu}\Psi + \left(\mathbb{X}\partial_{\lambda}\Phi\partial^{\lambda}\Phi + \mathbb{Y}\,\Box\,\Phi\right)\Psi + \mathbb{Z}\Psi = 0 \tag{30}$$

where

$$\mathbb{Z} \equiv \frac{2\alpha_0\Lambda V'}{Q}$$

$$\mathbb{X} = \frac{V'(\Phi H' - 2H - 6V'')}{Q} - H'$$

$$\mathbb{Y} = \frac{V'(2H\Phi - 6V')}{Q} - 2H$$

where $Q \equiv \alpha_0 + 2V + \Phi V'$.

At this stage, it is worth selecting among all possible candidates of V and H the particular ones that make the factor on the gradient and on $\Box$ of the field to disappear from equation (30). The simplest way is to set $\mathbb{X} = \mathbb{Y} = 0$ which imply only one condition, that is

$$H = \frac{-3(V')^2}{\alpha_0 + 2V}. \quad (31)$$

The non-minimal term V is such that $\mathbb{Z}$ reduces to a constant, that is

$$V = \frac{\alpha_0}{2}[(1+\sigma\Phi)^{-2} - 1]. \quad (32)$$

Then, it follows immediately that

$$H = -3\alpha_0\sigma^2(1+\sigma\Phi)^{-4} \quad (33)$$

where σ is a constant.

The equation for the spinor becomes

$$\mathrm{i}\gamma^\mu\nabla_\mu\Psi - m\Psi = 0 \quad (34)$$

where

$$m = \frac{4\,\sigma\Lambda}{\kappa\,c^2}. \quad (35)$$

Thus, as a result of the above process, the spinor field acquires a mass m that depends crucially on the existence of Λ. If Λ vanishes, then the mass of the field vanishes. The non-minimal coupling of gravity with the spinor field corresponds to a specific self-interaction. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe—represented by the cosmological constant. The values of different masses for different fields are contemplated in the parameter σ.

The various steps of our mechanism can be synthesized as follows.

- The dynamics of a massless spinor field Ψ is described by the Lagrangian
$$L_D = \frac{\mathrm{i}}{2}\bar\Psi\gamma^\mu\nabla_\mu\Psi - \frac{\mathrm{i}}{2}\nabla_\mu\bar\Psi\gamma^\mu\Psi.$$
- Gravity is described in general relativity by the scalar of curvature
$$L_E = R.$$
- The field interacts with gravity in a non-minimal way described by the term
$$L_{\mathrm{int}} = V(\Phi)R$$
where $\Phi = \bar\Psi\Psi$.
- The action of the ROTU on the spinor field, through the gravitational intermediary, is contained in the form of an additional constant term on the Lagrangian noted as Λ.
- A counter-term depending on the invariant Φ is introduced to kill extra terms coming from gravitational interaction and to avoid super-luminous propagation.
- As a result of this process, after specifying V and H, the field acquires a mass being described as
$$\mathrm{i}\gamma^\mu\nabla_\mu\Psi - m\Psi = 0$$
where m is given by equation (35) and is zero only if the cosmological constant vanishes.

This procedure allows us to state that the mechanism proposed here is to be understood as a realization of the Mach principle according to which the inertia of a body depends on the background of the ROTU. This strategy can be applied in a more general context in support of the idea that (local) properties of microphysics may depend on the (global) properties of the universe. We will analyze this in the next section.

Thus, collecting all these terms, we obtain the final form of the Lagrangian

$$L = \frac{\mathrm{i}}{2}\bar\Psi\gamma^\mu\nabla_\mu\Psi - \frac{\mathrm{i}}{2}\nabla_\mu\bar\Psi\gamma^\mu\Psi + \frac{1}{\kappa}(1+\sigma\Phi)^{-2}R - \frac{1}{\kappa}\Lambda - \frac{6}{\kappa}\sigma^2(1+\sigma\Phi)^{-4}\partial_\mu\Phi\partial^\mu\Phi. \quad (36)$$

2.2. Some comments

- In the case $\sigma = 0$, the Lagrangian reduces to a massless fermion satisfying Dirac's dynamics plus the gravitational field described by general relativity.
- The dimensionality of σ is L^3.
- The ratio $m/\sigma = 4\Lambda/\kappa\, c^2$ which has the meaning of a density of mass is an universal constant. How to interpret such universality?

A possible solution of this question can be found in the following way. The present mechanism is a real possibility if the mass obtained through it depends neither on the intensity of the gravitational field nor on Newton's constant, as it occurs in the previous case of the scalar field and for the vector field as it will be shown in the next section. In the case of fermions, we have found that the constant of interaction has dimensionality length[3]. Thus, one should write $\sigma = \sigma_0\, L^2_{\mathrm{Planck}} \Lambda^{-1/2}$ where σ_0 is a constant, obtaining in this case, for the mass of the fermions, the similar result $\mu = \sigma_0 \Lambda$.

3. The case of vector fields

We start with a scenario in which there are only three ingredients: a massless vector field, the gravitational field and an homogeneous distribution of energy—that are identified with the vacuum. The theory is specified by the Lagrangian

$$L = -\frac{1}{4}\, F_{\mu\nu}\, F^{\mu\nu} + \frac{1}{\kappa} R - \frac{\Lambda}{\kappa}. \tag{37}$$

The corresponding EOMs are

$$F^{\mu\nu}{}_{;\nu} = 0$$

and

$$\alpha_0 \big(R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} \big) = -T_{\mu\nu}$$

where $F_{\mu\nu} = \nabla_\nu W_\mu - \nabla_\mu W_\nu$ and $\alpha_0 \equiv 2/\kappa$.

In this theory, the vacuum Λ is invisible for W_μ. The energy distribution represented by Λ interacts with the vector field only indirectly, once it modifies the geometry of spacetime. In the Higgs mechanism, this vacuum is associated with a fundamental state of a scalar field φ and it is transformed in a mass term for W_μ. The role of Λ is displayed by the value of the potential $V(\varphi)$ in its homogeneous state. We will now show that there is no need to introduce any extra scalar field by using the universal character of gravitational interaction to generate mass for W_μ.

The point of departure is the recognition that gravity may be really responsible for breaking the gauge symmetry. For this, we modify the above Lagrangian to include a non-minimal coupling of the field W_μ to gravity in order to explicitly break such invariance. There are only two possible ways for this [2]. The total Lagrangian must be of the form

$$\mathbb{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{\kappa} R + \frac{\gamma}{6} R \Phi + \gamma\, R_{\mu\nu} W^\mu W^\nu - \frac{\Lambda}{\kappa} \tag{38}$$

where we define

$$\Phi \equiv W_\mu W^\mu.$$

The first two terms of $\mathbb{L}$ represent the free part of the vector and the gravitational fields. The third and fourth terms represent the non-minimal coupling interaction of the vector field with gravity. The parameter γ is dimensionless. The vacuum—represented by Λ—is added for the reasons presented above, and it must be understood as the definition of the expression 'the influence of the ROTU on W_μ'. We will not make any further hypothesis on this[5].

In the present proposed mechanism, Λ is the real responsible to provide mass for the vector field. This means that if we set $\Lambda = 0$ the mass of W_μ will vanish.

Independent variation of W_μ and $g_{\mu\nu}$ yields

$$F^{\mu\nu}{}_{;\nu} + \frac{\gamma}{3} R\, W^\mu + 2\gamma R^{\mu\nu} W_\nu = 0 \tag{39}$$

$$\alpha_0\left(R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu}\right) = -T_{\mu\nu}. \tag{40}$$

The energy–momentum tensor defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L)}{\delta g^{\mu\nu}}$$

is given by

$$\begin{aligned} T_{\mu\nu} = E_{\mu\nu} &+ \frac{\gamma}{3}\nabla_\mu\nabla_\nu\Phi - \frac{\gamma}{3}\Box\Phi g_{\mu\nu} + \frac{\gamma}{3}\Phi\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) \\ &+ \frac{\gamma}{3} R W_\mu W_\nu + 2\gamma R_\mu{}^\lambda W_\lambda W_\nu + 2\gamma R_\nu{}^\lambda W_\lambda W_\mu \\ &- \gamma R_{\alpha\beta} W^\alpha W^\beta g_{\mu\nu} - \gamma \nabla_\alpha\nabla_\beta (W^\alpha W^\beta) g_{\mu\nu} \\ &+ \gamma\nabla_\nu\nabla_\beta(W_\mu W^\beta) + \gamma\nabla_\mu\nabla_\beta(W_\nu W^\beta) + \gamma\,\Box(W_\mu W_\nu) + \frac{1}{\kappa}\Lambda g_{\mu\nu} \end{aligned} \tag{41}$$

where

$$E_{\mu\nu} = F_{\mu\alpha} F^\alpha{}_\nu + \tfrac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu}.$$

Taking the trace of equation (40), we obtain

$$R = 2\Lambda - \kappa\gamma\nabla_\alpha\nabla_\beta(W^\alpha W^\beta). \tag{42}$$

Then, using this result back in equation (39), it follows that

$$F^{\mu\nu}{}_{;\nu} + \frac{2\gamma\Lambda}{3} W^\mu - \frac{\kappa\gamma^2}{3}\nabla_\alpha\nabla_\beta(W^\alpha W^\beta)\, W^\mu + 2\gamma R^\mu{}_\nu W^\nu = 0. \tag{43}$$

The non-minimal coupling with gravity yields an effective self-interaction of the vector field and a term that represents its direct interaction with the curvature of spacetime. Besides, as a result of this process, the vector field acquires a mass μ that depends on the constant γ and on the existence of Λ. The term

$$2\gamma R^\mu{}_\nu W^\nu$$

gives a contribution (through the dynamics of the metric equation (40)) of $\gamma\Lambda$ yielding for the mass the formula

$$\mu^2 = \tfrac{5}{3}\gamma\Lambda. \tag{44}$$

Note that Newton's constant does not appear in our formula for the mass. The net effect of the non-minimal coupling of gravity with W^μ corresponds to a specific self-interaction of the vector field. The mass of the field appears only if we take into account the existence of the ROTU—represented by Λ—in the state in which this environment is in the corresponding vacuum. If Λ vanishes, then the mass of the field vanishes. The values of different masses for different fields are contemplated in the parameter γ.

[5] There is not any compelling reason to identify this constant with the actual cosmological constant or the value of the critical density 10^{-48} GeV^4 provided by cosmology.

3.1. Quantum perturbations

How this process that we have been examining here to give mass to all kind of bodies should be modified in a quantum version? We note, first of all, that the gravitational field will be treated at a classical level. Thus, any modification of the present scheme means to introduce quantum aspects of the vector field. This will not change the whole scheme of generation of mass described above. Indeed, in the semi-classical approach in which the matter field is quantized but the metric is not, the modification of the equation of general relativity becomes

$$\alpha_0\big(R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu}\big) = -\langle T_{\mu\nu}\rangle \tag{45}$$

where the field is in a given specific state. Throughout all the processes of gravitational interaction, the system does not change its state, allowing the same classical treatment as above.

4. The case of spin-2 field

As in the previous cases, we start with a scenario in which there are only three ingredients: a linear tensor field, the gravitational field and a homogeneous distribution of energy identified with the vacuum. We note that there are two possible equivalent ways to describe a spin-2 field that is

- Einstein frame (EF) and
- Fierz frame (FF).

Accordingly, we use a symmetric second-order tensor $\varphi_{\mu\nu}$ or the third-order tensor $F_{\alpha\beta\lambda}$. Although the Fierz representation is not used for most of the work dealing with spin-2 field, it is far better than the EF when dealing in a curved spacetime [5]. Thus, let us review briefly the basic properties of the FF[6]. We start by defining a three-index tensor $F_{\alpha\beta\mu}$ which is anti-symmetric in the first pair of indices and obeys the cyclic identity:

$$F_{\alpha\mu\nu} + F_{\mu\alpha\nu} = 0, \tag{46}$$

$$F_{\alpha\mu\nu} + F_{\mu\nu\alpha} + F_{\nu\alpha\mu} = 0. \tag{47}$$

This expression implies that the dual of $F_{\alpha\mu\nu}$ is trace-free:

$$\overset{*}{F}{}^{\alpha\mu}{}_{\mu} = 0, \tag{48}$$

where the asterisk represents the dual operator, defined in terms of $\eta_{\alpha\beta\mu\nu}$ by

$$\overset{*}{F}{}^{\alpha\mu}{}_{\lambda} \equiv \tfrac{1}{2}\eta^{\alpha\mu}{}_{\nu\sigma}F^{\nu\sigma}{}_{\lambda}.$$

The tensor $F_{\alpha\mu\nu}$ has 20 independent components. The necessary and sufficient condition for $F_{\alpha\mu\nu}$ to represent a unique spin-2 field (described by ten components) is[7]

$$\overset{*}{F}{}^{\alpha(\mu\nu)}{}_{,\alpha} = 0, \tag{49}$$

which can be rewritten as

$$F_{\alpha\beta}{}^{\lambda}{}_{,\mu} + F_{\beta\mu}{}^{\lambda}{}_{,\alpha} + F_{\mu\alpha}{}^{\lambda}{}_{,\beta} - \tfrac{1}{2}\delta^{\lambda}_{\alpha}(F_{\mu,\beta} - F_{\beta,\mu})$$
$$- \tfrac{1}{2}\delta^{\lambda}_{\mu}(F_{\beta,\alpha} - F_{\alpha,\beta}) - \tfrac{1}{2}\delta^{\lambda}_{\beta}(F_{\alpha,\mu} - F_{\mu,\alpha}) = 0. \tag{50}$$

[6] We use the notations $A_{(\alpha}B_{\beta)} = A_\alpha B_\beta + A_\beta B_\alpha$, and $A_{[\alpha}B_{\beta]} = A_\alpha B_\beta - A_\beta B_\alpha$.

[7] Note that this condition is analogous to that necessary for the existence of a potential A_μ for the electromagnetic field, given by $\overset{*}{A}{}^{\alpha\mu}{}_{,\alpha} = 0$.

A direct consequence of the above equation is the identity

$$F^{\alpha\beta\mu}{}_{,\mu} = 0. \tag{51}$$

We call a tensor that satisfies the conditions given in equations (46), (47) and (49) a Fierz tensor. If $F_{\alpha\mu\nu}$ is a Fierz tensor, it represents a unique spin-2 field. Condition (49) yields a connection between the EF and the FF: it implies that there exists a symmetric second-order tensor $\varphi_{\mu\nu}$ that acts as a potential for the field. We write

$$2\,F_{\alpha\mu\nu} = \varphi_{\nu[\alpha,\mu]} + \left(\varphi_{,\alpha} - \varphi_{\alpha}{}^{\lambda}{}_{,\lambda}\right)\,\eta_{\mu\nu} - \left(\varphi_{,\mu} - \varphi_{\mu}{}^{\lambda}{}_{,\lambda}\right)\,\eta_{\alpha\nu}, \tag{52}$$

where $\eta_{\mu\nu}$ is the flat spacetime metric tensor, and the factor 2 on the lhs is introduced for convenience.

Taking the trace of equation (52) $F_{\alpha} \equiv F_{\alpha\mu\nu}\eta^{\mu\nu}$, it follows that

$$F_{\alpha} = \varphi_{,\alpha} - \varphi_{\alpha}{}^{\lambda}{}_{,\lambda},$$

where φ is the trace φ^{μ}_{μ}. Thus, we can write

$$2F_{\alpha\mu\nu} = \varphi_{\nu[\alpha,\mu]} + F_{[\alpha}\,\eta_{\mu]\nu}. \tag{53}$$

Using the properties of the Fierz tensor, we obtain the important identity

$$F^{\alpha}{}_{(\mu\nu),\alpha} \equiv -2G^{(L)}{}_{\mu\nu}, \tag{54}$$

where $G^{(L)}{}_{\mu\nu}$ is the linearized Einstein tensor, defined by the perturbation $g_{\mu\nu} = \eta_{\mu\nu} + \varphi_{\mu\nu}$ as

$$2G^{(L)}{}_{\mu\nu} \equiv \Box\,\varphi_{\mu\nu} - \varphi^{\epsilon}{}_{(\mu,\nu),\epsilon} + \varphi_{,\mu\nu} - \eta_{\mu\nu}\left(\Box\,\varphi - \varphi^{\alpha\beta}{}_{,\alpha\beta}\right). \tag{55}$$

The divergence of $F^{\alpha}{}_{(\mu\nu),\alpha}$ yields the Bianchi identity

$$F^{\alpha(\mu\nu)}{}_{,\alpha\mu} \equiv 0. \tag{56}$$

Indeed,

$$F^{\alpha\mu\nu}{}_{,\alpha\mu} + F^{\alpha\nu\mu}{}_{,\mu\alpha} = 0. \tag{57}$$

The first term vanishes identically due to the symmetric properties of the field and the second term vanishes due to equation (51). Using equation (54), the identity which states that the linearized Einstein tensor $G^{(L)}{}_{\mu\nu}$ is divergence free is recovered.

We shall build now dynamical equations for the free Fierz tensor in flat spacetime. Our considerations will be restricted here to linear dynamics. The most general theory can be constructed from a combination of the three invariants involving the field. These are represented by A, B and W as follows:

$$A \equiv F_{\alpha\mu\nu}\,F^{\alpha\mu\nu}, \qquad B \equiv F_{\mu}\,F^{\mu},$$

$$W \equiv F_{\alpha\beta\lambda}\,\overset{*}{F}{}^{\alpha\beta\lambda} = \tfrac{1}{2}F_{\alpha\beta\lambda}\,F^{\mu\nu\lambda}\eta^{\alpha\beta}{}_{\mu\nu}.$$

Here, W is a topological invariant so we shall use only the invariants A and B. The EOM for the massless spin-2 field in the ER is given by

$$G^{(L)}{}_{\mu\nu} = 0. \tag{58}$$

As we have seen above, in terms of the field $F^{\lambda\mu\nu}$, this equation can be written as

$$F^{\lambda(\mu\nu)}{}_{,\lambda} = 0. \tag{59}$$

The corresponding action takes the form

$$S = \frac{1}{k}\int \mathrm{d}^4x\,(A - B). \tag{60}$$

Then,

$$\delta S = \int F^{\alpha(\mu\nu)}{}_{,\alpha}\, \delta\varphi_{\mu\nu}\, \mathrm{d}^4x. \tag{61}$$

We obtain

$$\delta S = -2 \int G^{(L)}{}_{\mu\nu}\, \delta\varphi^{\mu\nu}\, \mathrm{d}^4x, \tag{62}$$

where $G^{(L)}{}_{\mu\nu}$ is given in equation (55).

Let us consider now the massive case. If we include a mass for the spin-2 field in the FF, the Lagrangian takes the form

$$\mathcal{L} = A - B + \frac{m^2}{2}(\varphi_{\mu\nu}\varphi^{\mu\nu} - \varphi^2), \tag{63}$$

and the EOMs that follow are

$$F^{\alpha}{}_{(\mu\nu),\alpha} - m^2(\varphi_{\mu\nu} - \varphi\eta_{\mu\nu}) = 0, \tag{64}$$

or equivalently,

$$G^{(L)}{}_{\mu\nu} + \frac{m^2}{2}(\varphi_{\mu\nu} - \varphi\eta_{\mu\nu}) = 0.$$

The trace of this equation gives

$$F^{\alpha}{}_{,\alpha} + \tfrac{3}{2}\, m^2\varphi = 0, \tag{65}$$

while the divergence of equation (64) yields

$$F_{\mu} = 0. \tag{66}$$

This result together with the trace equation gives $\varphi = 0$.

In terms of the potential, equation (66) is equivalent to

$$\varphi_{,\,\mu} - \varphi^{\epsilon}{}_{\mu\,,\epsilon} = 0. \tag{67}$$

It follows that we must have

$$\varphi^{\mu\nu}{}_{,\nu} = 0.$$

Thus, we have shown that the original ten degrees of freedom (DOF) of $F_{\alpha\beta\mu}$ have been reduced to five (which is the correct number for a massive spin-2 field) by means of the five constraints

$$\varphi^{\mu\nu}{}_{,\nu} = 0, \qquad \varphi = 0. \tag{68}$$

4.1. Equation of spin-2 in curved background

The passage of the spin-2 field equation from Minkowski spacetime to arbitrary curved Riemannian manifold presents ambiguities due to the presence of second-order derivatives of the rank 2 symmetric tensor $\varphi_{\mu\nu}$ that is used in the so-called EF (see for instance [5]). These ambiguities disappear when we move to the FF representation that deals with the three-index tensor $F_{\alpha\mu\nu}$ as was shown in [5].

There results yield a unique form of minimal coupling, free of ambiguities. Let us define from $\varphi_{\mu\nu}$ two auxiliary fields $G^{(\mathrm{I})}{}_{\mu\nu}$ and $G^{(\mathrm{II})}{}_{\mu\nu}$ through the expressions

$$2G^{(\mathrm{I})}{}_{\mu\nu} \equiv \Box\,\varphi_{\mu\nu} - \varphi_{\epsilon(\mu;\nu)}{}^{;\epsilon} + \varphi_{;\mu\nu} - \eta_{\mu\nu}\left(\Box\,\varphi - \varphi^{\alpha\beta}{}_{;\alpha\beta}\right), \tag{69}$$

$$2G^{(\mathrm{II})}{}_{\mu\nu} \equiv \Box\,\varphi_{\mu\nu} - \varphi_{\epsilon(\mu}{}^{;\epsilon}{}_{;\nu)} + \varphi_{;\mu\nu} - \eta_{\mu\nu}\left(\Box\,\varphi - \varphi^{\alpha\beta}{}_{;\alpha\beta}\right). \tag{70}$$

These objects differ only in the order of the second derivative in the second term on the rhs of the above equations. The EOM (63) free of ambiguities concerns the tensor field

$$\widehat{G}_{\mu\nu} \equiv \tfrac{1}{2}(G^{(\mathrm{I})}{}_{\mu\nu} + G^{(\mathrm{II})}{}_{\mu\nu}) \tag{71}$$

and is given by

$$\widehat{G}_{\mu\nu} + \tfrac{1}{2}\,m^2(\varphi_{\mu\nu} - \varphi g_{\mu\nu}) = 0. \tag{72}$$

which is precisely the usual equations for the massive spin-2 field.

4.2. Generating mass for the spin-2 field

We follow the same strategy as in the previous case and take the dynamics of the spin-2 field as given by

$$\mathbb{L} = F_{\alpha\mu\nu}\,F^{\alpha\mu\nu} - F_\alpha\,F^\alpha + \frac{1}{\kappa}R + a\,R_{\alpha\mu\beta\nu}\varphi^{\alpha\beta}\varphi^{\mu\nu} - \frac{\Lambda}{\kappa}. \tag{73}$$

The EOMs are given by

$$F^\alpha{}_{(\mu\nu);\alpha} + 2a\,R_{\alpha\mu\beta\nu}\varphi^{\alpha\beta} = 0, \tag{74}$$

$$\frac{1}{\kappa}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{\Lambda}{2}g_{\mu\nu}\right) + T_{\mu\nu} + a\,Y_{\mu\nu} = 0 \tag{75}$$

where the quantity $Y_{\mu\nu}$ is given by the variation of the non-minimal coupling term

$$\delta\int\sqrt{-g}\,R_{\alpha\mu\beta\nu}\varphi^{\alpha\beta}\varphi^{\mu\nu} = \int\sqrt{-g}\,Y_{\mu\nu}\delta g_{\mu\nu}. \tag{76}$$

Here, $Y_{\mu\nu}$ is given in terms of $S_{\alpha\mu\beta\nu}$ defined as

$$S_{\alpha\mu\beta\nu} \equiv \varphi_{\alpha\beta}\varphi_{\mu\nu} - \varphi_{\alpha\nu}\varphi_{\beta\mu}$$

which has the symmetries

$$S_{\alpha\mu\beta\nu} = -S_{\alpha\mu\nu\beta} = -S_{\mu\alpha\beta\nu} = S_{\beta\nu\alpha\mu}.$$

A direct calculation yields

$$Y^{\mu\nu} \equiv S^{\lambda\mu\nu\epsilon}{}_{;\epsilon;\lambda} - \tfrac{1}{2}R_{\alpha\sigma\beta\lambda}\varphi^{\alpha\beta}\varphi^{\sigma\lambda}g^{\mu\nu} + \tfrac{3}{2}R_{\alpha\sigma\beta}{}^{(\mu}\varphi^{\nu)\sigma}\varphi^{\alpha\beta} - \tfrac{1}{2}R_{\alpha\sigma\beta\lambda}\varphi^{\alpha\beta}\varphi^{\sigma\lambda}g_{\mu\nu}. \tag{77}$$

Let us recall that the Riemann curvature can be written in terms of its irreducible quantities involving the Weyl conformal tensor $W_{\alpha\sigma\beta\lambda}$ and the contracted Ricci tensor by the formula

$$R_{\alpha\mu\beta\nu} = W_{\alpha\mu\beta\nu} + \tfrac{1}{2}\left(R_{\alpha\beta}g_{\mu\nu} + R_{\mu\nu}g_{\alpha\beta} - R_{\alpha\nu}g_{\beta\mu} - R_{\beta\mu}g_{\alpha\nu}\right) - \tfrac{1}{6}Rg_{\alpha\mu\beta\nu}. \tag{78}$$

Then,

$$R_{\alpha\mu\beta\nu}\varphi^{\alpha\beta}\varphi^{\mu\nu} = W_{\alpha\mu\beta\nu}\varphi^{\alpha\beta}\varphi^{\mu\nu} + \left(R_{\alpha\beta} - \tfrac{1}{6}Rg_{\alpha\beta}\right)(\varphi\varphi^{\alpha\beta} - \varphi^\alpha{}_\lambda\varphi^{\lambda\beta}). \tag{79}$$

We can then rewrite the equation of the spin-2 field as

$$F^\alpha{}_{(\mu\nu);\alpha} - \frac{a\Lambda}{3}(\varphi_{\mu\nu} - \varphi g_{\mu\nu}) + 2a\,W_{\alpha\mu\beta\nu}\varphi^{\alpha\beta} + Q_{\mu\nu} = 0, \tag{80}$$

where $Q_{\mu\nu}$ contains the nonlinear terms of the interaction of the spin-2 field with gravity. A comparison with equation (64) yields the mass

$$m^2 = \frac{a\Lambda}{3}.$$

Note that Newton's constant does not appear in this formula for the mass. The net effect of the non-minimal coupling of gravity with $F^{\alpha\beta\mu}$ corresponds to a specific self-interaction of the tensor field. The mass of the field appears only if we take into account the existence of the ROTU—represented by Λ—in the state in which this environment is in the corresponding vacuum. If Λ vanishes, then the mass of the field vanishes. The values of different masses for different fields are contemplated in the dimensionless parameter a.

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