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Repulsive gravity generated by ordinary matter

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Abstract

In the present work, and idea first discussed in the early 80's [**? ? ?**] is rediscussed. The model consists of a scalar field non-minimally coupled to gravity with a quartic self-interaction potential. After a spontaneous symmetry braking process, the scalar field promotes a renormalization of the gravitational constant, which in certain regimes can lead to gravitational repulsive effects produced by ordinary matter (more specifically, radiation). No exotic potential is required by this mechanism, and the broken symmetry can only last during a radiation dominated phase of the universe.

Keywords: Scalar field cosmology, spontaneous symmetry breaking, non-minimal coupling

I. INTRODUCTION

Gravity is a strictly atractive force. And yet, repulsive gravitational effects are a central theme in the Stadard Cosmological Model (SCM), where they are evoked to overcome the difficulties faced by the Friedmann Cosmology — the horizon, flatness, size of the universe, and origin of the primordial density fluctuations problems (Guth [**?**], see ... for earlier ideas), among others, as well as the observed late-time accelerated expansion of the universe in connection with the cosmological constant problem (Riess *et al.* [**?**], Perlmutter *et al.* [**?**], Weinberg [**?**]). Two distinct phases of accelerated cosmic expansion are thus expected according to the SCM, each with a different "gravitational repulsion" mechanism behind. The first three problems are overcome by postulating the existence of a cosmic inflation phase in the early universe, while the last is adressed by assuming the universe today is dominated by a (misterious) dark energy, reminiscent of the oldest gravitational repulsion mechanism known, Einstein's cosmological constant Λ.

According to General Relativity (GR), in order to produce an accelerated expansion phase at some point in the cosmic history, the matter content of the universe must be dominated by a fluid with negative pressure at that point. The natural candidate for such a fluid, the vacuum eneregy density of the matter fields, has a theoretical value approximatelly 120 orders of magnitude bigger then the one associated to the observed effects. Moreover, such an effect would last forever, being thus in conflict wuth structure formation models.

Scalar fields, on the other hand, are the only type of field admiting a potential term in their Lagrangian capable of producing a negative pressure on certain regimes. The Higgs model provided the prototypical field theory with the needed properties, a scalar field with dynamical properties modulated by a spontaneous symmetry breaking process. In the oldest inflationary models, first order phase transitions of Grand Unified Theories (GUT) let to an exponential expansion of the universe. In those models, however, the necessary conditions for the inflation to start was set by artificially fine-tuned initial conditions, while the scalar field's potential had to be sufficiently flat around the metastable maximum (false vacuum). In the new inflationary scenarios, on the other hand, the initial conditions problem was overcome the slow-roll inflationary mechanism. Again, the potential need to be sufficiently flat to garantee the slow-roll conditions. All those models require the existence of new real (neutral) scalar fields with "exotic" self interaction potentials.

So, the question arises: can ordinary matter generate gravitational repulsion effects? As will be shown in what follows, ordinary matter can have such an extraordinary behaviour in certain special circunstances.

The main idea revisited here was first presented in a series of publications back in the early 80's (Novello [**? ? ?**]). The reason to revisit it now is twofold, first to ameliorate the results and the discussion presented there[1], and second to restate the implications of this idea for what became know later as the inflationary paradigm (see Guth [**?**]).

II. MODEL

A. Non-minimally Coupled Scalar Field

In what follows, we will adopt the space-time metric's signature $(+ - --)$. The theory considered here is defined by the Lagrangian with a scalar field non-minimally coupled to gravity

$$L = \sqrt{-g}\left[\frac{1}{2\kappa}R + \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - V(\phi) - \frac{1}{12}R\phi^2\right], \tag{1}$$

where $g = \det(g_{\alpha\beta})$, R is the Ricci scalar, $\kappa = 8\pi G$ is the reduced gravitational constant, and the self-interaction potential of the scalar field is given by

$$V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{1}{2}\sigma\phi^4 - V_0, \tag{2}$$

$m^2 > 0$ and $V_0 > 0$ being assumed. A real scalar field was choosen only for the sake of simplicity. We shall take the (reduced) gravitational constant equal to one from now on ($\kappa = 8\pi G = c = \hbar = 1$).

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[1] There was a mistake in the field equations appearing in those papers which passed unnoticed until today. That mistake, however, had no impact on the main result presented there, and restated here, which is the possibility of an effective gravitational repulsion generated by ordinary matter on certain circunstances. That mistake was corrected here.

The variation of the Lagrangia (**??**) with respect to the metric and the scalar field produces the set of field equations

$$\left(1-\frac{1}{6}\phi^2\right)\left(R_{\alpha\beta}-\frac{1}{2}Rg_{\alpha\beta}\right)=-\,\partial_\alpha\phi\partial_\beta\phi+\frac{1}{2}\Big(\partial_\mu\phi\partial^\mu\phi-m^2\phi^2+\sigma\phi^4\Big)g_{\alpha\beta}+\frac{1}{6}\Big(\nabla_\alpha\nabla_\beta\phi^2-\Box\phi^2 g_{\alpha\beta}\Big)$$
$$+V_0 g_{\alpha\beta}, \tag{3a}$$

$$\Box\phi+m^2\phi-2\sigma\phi^3+\frac{1}{6}R\phi=0, \tag{3b}$$

where $\Box\equiv g^{\alpha\beta}\nabla_\alpha\nabla_\beta$. Taking the trace of the field equation (**??**), and using equation (**??**), it follows that the Ricci scalar can be expressed as

$$R=m^2\phi^2-4V_0. \tag{4}$$

This enables to rewrite equation (**??**) in terms of the potential $V(\phi)$ alone as follows

$$\Box\phi+\left(m^2-\frac{2}{3}V_0\right)\phi+\left(\frac{1}{6}m^2-2\sigma\right)\phi^3=0. \tag{5}$$

B. Spontaneous Symmetry Breaking

It is clear from equation (**??**) that in the case where the ground state of the scalar field potential vanishes, $V_0=0$, the energy-momentum tensor of the scalar field

$$t_{\alpha\beta}(\phi)\equiv\partial_\alpha\phi\partial_\beta\phi-\frac{1}{2}\Big(\partial_\mu\phi\partial^\mu\phi-m^2\phi^2+\sigma\phi^4\Big)g_{\alpha\beta}-\frac{1}{6}\Big(\nabla_\alpha\nabla_\beta\phi^2-\Box\phi^2 g_{\alpha\beta}\Big) \tag{6}$$

is irremediably not conserved. Alternatively, the gravitational field equation (**??**) can be rewriten in the form

$$R_{\alpha\beta}-\frac{1}{2}Rg_{\alpha\beta}=-\,E_{\alpha\beta}(\phi), \tag{7}$$

where we have defined the energy-momentum complex

$$E_{\alpha\beta}(\phi)\equiv\frac{6t_{\alpha\beta}(\phi)-6V_0 g_{\alpha\beta}}{6-\phi^2}. \tag{8}$$

It is clear that the energy-momentum complex (**??**) can now be conserved ([**?**]).

We now look for a constant solutions of equation (**??**) which correspond to stable vacua of the scalar field. According to expressions (**??**)-(**??**), for a constant solution $\phi=\phi_0$ the energy density $E_{00}(\phi)=E(\phi)$ corresponding to the energy-momentum complex (**??**) has the form

$$E(\phi_0)=\frac{3m^2\phi_0^2-3\sigma\phi_0^4-6V_0}{6-\phi_0^2}. \tag{9}$$

Equation (**??**), on the other hand, admits only two constant solutions, *viz.* $\phi_0=0$ and

$$\phi_0^2=\frac{6m^2-4V_0}{12\sigma-m^2}, \tag{10}$$

The constant nontrivial solutions which minimizes the energy density functional (**??**), and satisfies relation (**??**) is given by

$$\phi_0=\pm\frac{2\sqrt{V_0}}{m},\qquad \sigma=\frac{m^4}{8V_0}. \tag{11}$$

The resulting behaviour of the energy density functional $E(\phi)$ exhibits three uncommunicating regions, two of them containing the stable local minima at $\phi=\pm2\sqrt{V_0}/m$, the other containing two unstable maxima and a metastable minimum at $\phi=0$ (see Fig **??**).

FIG. 1: Plot of the energy functional $E(\phi)$, for $V_0 > 3m^2/2 > 0$. The figure shows the solutions $\phi_0 = \pm 2\sqrt{V_0}/m$ corresponding to the nontrivial stable vacua. The graphic has two asymptotes at $\phi = \pm\sqrt{6}$ separating three uncommunicating regions.

FIG. 2: Plot of the energy functional $E(\phi)$, for different values of σ.

FIG. 3: Plot of the energy functional $E(\phi)$, for different values of V_0.

On the other hand, the field equation (**??**) for the constant solution $\phi = \phi_0$ assumes the form

$$\left(1 - \frac{1}{6}\phi_0^2\right)\left(R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta}\right) = -\frac{1}{2}V(\phi_0)g_{\alpha\beta} \tag{12}$$

Taking the trace of equation (**??**) and using it back in that equation, we can rewrite it as follows

$$R_{\alpha\beta} = \left(\frac{3m^2\phi_0^2 - 3\sigma\phi_0^4 - 6V_0}{6 - \phi_0^2}\right)g_{\alpha\beta}\,. \tag{13}$$

As a consequence, it follows that for the particular solution (**??**), and in the absence of matter fields, the gravitational field equations are exactly the vacuum equations of GR

$$R_{\alpha\beta} = 0. \tag{14}$$

C. Repulsive Gravitational Effects from Ordinary Matter

Like in the Higgs model, where a spontaneous symmetry breaking mechanism promotes a "restructuring" of the vacuum, in the present model a similar spontaneous symmetry breaking process of the scalar field induced by the vacuum has the effect of producing an accelerated expantion of the universe. This situation constrasts with the standard inflationary scenarios, where a slow-roll process of an inflaton field with a flat potential is required for inflation.

Matter fields can be introduced in the model in a straigthfoward way. The gravitational field equations in the presence of matter fields assume the form

$$\left(1-\frac{1}{6}\phi^2\right)\left(R_{\alpha\beta}-\frac{1}{2}Rg_{\alpha\beta}\right)=-t_{\alpha\beta}(\phi)-T_{\alpha\beta}+V_0g_{\alpha\beta}, \tag{15}$$

where $T_{\alpha\beta}$ is the energy-momentum tensor of the matter fields. Taking the trace of the field equation (**??**), and again using equation (**??**), we obtain the expression of the Ricci scalar

$$R=m^2\phi^2-4V_0+T, \tag{16}$$

where $T\equiv g^{\alpha\beta}T_{\alpha\beta}$. This enables us to rewrite equation (**??**) in the form

$$\Box\phi+\left(m^2-\frac{2}{3}V_0+\frac{1}{6}T\right)\phi+\left(\frac{1}{6}m^2-2\sigma\right)\phi^3=0. \tag{17}$$

Clearly, equation (**??**) only admits a constant solution in the special case where the trace of the energy momentum-tensor of matter fields is a constant. We stress the fact that the spontaneous symmetry breaking process is only possible in this case, otherwise no nontrivial ground state solution of equation (**??**) exists. Let us concentrate, for now, on matter fields described by an energy-momentum tensor with null trace, $T=0$ (radiation).

Similarly to what was done in the presvious section, we can rewrite the gravitational field equations in an analogous form

$$R_{\alpha\beta}-\frac{1}{2}Rg_{\alpha\beta}=-E_{\alpha\beta}(\phi)-\left(\frac{6}{6-\phi^2}\right)T_{\alpha\beta}. \tag{18}$$

For $T=0$, the constant nontrivial solutions of equation (**??**) which extremizes the energy density $E(\phi)$ are again given by

$$\phi_0=\pm\frac{2\sqrt{V_0}}{m},\quad \sigma=\frac{m^4}{8V_0}. \tag{19}$$

Consequently, in the case where the trace of the energy-momentum tensor of matter fields vanishes, when the scalar field is in the groud state $\phi=\phi_0$ corresponding to one of its stable vacua, as given by equations (**??**), the gravitational field equation (**??**) can be finally written as

$$R_{\alpha\beta}=-\left(\frac{3m^2\kappa}{3m^2-2\kappa V_0}\right)T_{\alpha\beta}, \tag{20}$$

where, for the sake of clarity, the gravitational constant $\kappa=8\pi G$ was recovered. The term multiplying the energy-momentum tensor of ordinary matter (radiation) in equation (**??**) can, thus, be viewed as a "renormalized" gravitational constant. Clearly, in the regime where the relation

$$\kappa V_0>\frac{3}{2}m^2 \tag{21}$$

is satisfied, gravity is reversed.

D. Complex Scalar Field

$$L=\sqrt{-g}\left[\frac{1}{2\kappa}R+\partial_\alpha\Phi^*\partial^\alpha\Phi-V(\Phi^*\Phi)-\frac{1}{6}R\Phi^*\Phi\right], \tag{22}$$

III. COSMOLOGY

$$ds^2 = dt^2 - A^2(t)\left(\frac{dr^2}{1-\epsilon r^2} + r^2\, d\vartheta^2 + r^2 \sin^2\vartheta\, d\phi^2\right) , \tag{23}$$

$$\epsilon = +1\,,\,0\,,\,-1\,,$$

$$\left(\frac{\dot{A}}{A}\right)^2 + \frac{\epsilon}{A^2} = \frac{1}{3}\left(\frac{3m^2\kappa}{3m^2 - 2\kappa V_0}\right)\rho_R \tag{24}$$

$$\frac{\ddot{A}}{A} = -\frac{1}{3}\left(\frac{3m^2\kappa}{3m^2 - 2\kappa V_0}\right)\rho_R \tag{25}$$

In the case where $2\kappa V_0 > 3m^2$...

IV. FINAL REMARKS