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Toy model of a fake inflation

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Discontinuities in nonlinear field theories propagate through null geodesics in an effective metric that depends on its dynamics and on the background geometry. Once information of the geometry of the universe comes mostly from photons, one should carefully analyze the effects of possible nonlinearities on electrodynamics in the cosmic geometry. Such a phenomenon of induced metric is rather general and may occur for any nonlinear theory independently of its spin properties. We limit our analysis here to the simplest case of nonlinear scalar field. We show that a class of theories that have been analyzed in the literature, having regular configuration in the Minkowski space-time background, is such that the field propagates like free waves in an effective de Sitter geometry. The observation of these waves would lead us to infer, erroneously, that we live in a de Sitter universe.

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I. INTRODUCTION

We learned from general relativity that the geometry of space-time is guided by gravitational forces. The possibility of such identification of gravity with geometry relies on the universality of such interaction. Nevertheless, in certain cases, dealing with a not so general interaction, it is worth describing certain kinds of evolutionary processes by appealing to an effective modification of the geometry. This is the case, for instance, with the propagation of waves of spin zero (scalarons), spin one (photons) in nonlinear field theories, and the sonic analogue of black holes [1–3]. Indeed, it was shown in these papers that the discontinuities of nonlinear theories propagate through curves which are null geodesics of an effective geometry $\hat{g}_{\mu\nu}$ which depends not only on the dynamics but also on the properties of the background field.

The importance of such analogue models, dealing with modifications of the geometry which are not consequences of gravitational processes, is related to the impossibility to control gravitational fields in laboratory experiments. The fact that we can, in principle, produce specific cases of geometries which have similar properties of solutions of the equations of general relativity, allows us to understand a little better, the behavior of matter in gravity interaction by the analysis of analogous situations, using others interactions, which are capable to be under our experimental control. The case of the emission of radiation by a black hole is a typical one, once it is understood [4] that a similar behavior could occur either in sonic or in electromagnetic black holes [3].

Such an effective description allows us to pose the following question: is it possible, for a given field theory to exhibit a configuration, satisfying nonlinear equations of motion in Minkowski background and satisfying the property that the propagation of the waves of the field experience in this state a prescribed geometry, to be specific, e.g., the one described by de Sitter?

In this letter we show that the answer is positive, and we exhibit an example that corresponds to a situation in which it occurs. In order to simplify our calculation, we consider the case of a nonlinear scalar field configuration [5]. The reason for this is twofold: it is the simplest case to deal with and it constitutes a fundamental element of the scenario that cosmologists are using nowadays as viable candidates to represent the basic ingredient of the matter content of the universe, that is, dark energy. According to this last motivation, our study here can be understood as a toy model for a fake inflation.

II. THE NONLINEAR DYNAMICS OF A SCALAR FIELD

The observation of the acceleration of the universe has brought into attention new candidates to describe forms of matter with some unusual properties. One of these is the so-called Chaplygin gas [6,7]. A remarkable property of this fluid is that its energy content can be equivalently described in terms of a scalar field that satisfies a nonlinear dynamics obtained from the Born-Infeld action. A certain number of distinct models of nonlinear theories is being studied. The important point which is relevant for our analysis concerns the propagation of the associated scalar waves.

We consider a class of Lagrangians [8] of the form $\mathcal{L}(w, \varphi) = f(w) - V(\varphi)$, where $w := \partial_\mu \varphi \partial^\mu \varphi$. The first and second derivatives of $\mathcal{L}$ with respect to w are denoted L_w and L_{ww}, respectively. The equation of motion for φ reads

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$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}L_w(\partial_\nu\varphi)) = -\frac{1}{2}\frac{\delta V}{\delta\varphi}. \tag{1}$$

We are interested in evaluating the characteristic surfaces of wave propagation of this theory. The most direct and elegant way to pursue this goal is to use the Hadamard formalism [9–12]. Let Σ be a surface of discontinuity of the scalar field φ. We define the discontinuity of an arbitrary function f to be given by

$$[f(x)]|_\Sigma = \lim_{\epsilon\to 0^+}(f(x+\epsilon) - f(x-\epsilon)). \tag{2}$$

We take that φ and its first derivative $\partial_\mu\varphi$ are continuous across Σ, while the second derivatives present a discontinuity:

$$[\varphi(x)]|_\Sigma = 0, \tag{3}$$

$$[\partial_\mu\varphi(x)]|_\Sigma = 0, \tag{4}$$

$$[\partial_\mu\partial_\nu\varphi(x)]|_\Sigma = k_\mu k_\nu \xi(x), \tag{5}$$

where $k_\mu := \partial_\mu\Sigma$ is the propagation vector and $\xi(x)$ the amplitude of the discontinuity. Once $\frac{\delta V}{\delta\varphi}$ is continuous across Σ and applying (3) to (1) we find

$$k_\mu k_\nu(L_w g^{\mu\nu} + 2L_{ww}\partial^\mu\varphi\partial^\nu\varphi) = 0.$$

This equation suggests the introduction of an effective metric defined by

$$\hat{g}^{\mu\nu} := L_w g^{\mu\nu} + 2L_{ww}\partial^\mu\varphi\partial^\nu\varphi. \tag{6}$$

The inverse $\hat{g}_{\mu\nu}$ of (6) is obtained by using the ansatz $\hat{g}_{\mu\nu} = Ag_{\mu\nu} + B\partial_\mu\varphi\partial_\nu\varphi$ where the unknown coefficients A and B are determined through the condition $\hat{g}^{\mu\alpha}\hat{g}_{\alpha\nu} = \delta^\mu_\nu$. This leads to

$$\hat{g}_{\mu\nu} := \frac{1}{L_w}\left(g_{\mu\nu} - \frac{2L_{ww}}{\Psi}\partial_\mu\varphi\partial_\nu\varphi\right), \tag{7}$$

where we defined $\Psi := L_w + 2wL_{ww}$.

III. METHODOLOGY

Since the scalar field "see" the effective geometry one can ask for nonlinear Lagrangians leading to a given effective geometry in a fixed background. To this end, one proceeds by choosing a Lagrangian, determining the corresponding effective geometry, and solving the Euler-Lagrange equations for φ. Unfortunately since the effective metric depends on the field, such an approach is often intractable. A convenient means to simplify the problem is to choose Ψ, this allows one to partly control the interplay between the Lagrangian and the effective geometry (7). Let us examine the simplest case where $\Psi = 1$ which we use hereafter. This choice obviously simplifies (7) and is equivalent to the equation: $L_w + 2wL_{ww} = 1$. This equation can be straightforwardly integrated to yield

$$\mathcal{L} = w + 2\lambda\sqrt{w} + C, \tag{8}$$

where λ is a nonzero c-number and C a constant with respect to w, in particular, one can set $C = -V(\varphi)$. This case is worth considering, in particular, because of the properties it provides for the effective metric, but besides this it can be understood as a perturbation of the standard linear theory. Just to present a toy model that corresponds to a specific "fake inflation" we will restrict the case in which the potential takes the form [13]

$$V(\varphi) = -\lambda^2\mathbf{x}\left(1 + \frac{\mathbf{x}}{2}\right), \tag{9}$$

where we have defined $\mathbf{x} \equiv e^{-(2H/\lambda)\varphi}$.

IV. EFFECTIVE FRW METRIC IN A MINKOWSKIAN BACKGROUND

We first set the background metric to the Minkowski metric $\eta_{\mu\nu}$, and use the convention $(+,-,-,-)$. We now show that for a field φ depending only on time $\varphi = \varphi(t)$ and satisfying the Lagrangian (8) the effective metric $\hat{g}$ felt by φ is a spatially flat Friedmann-Robertson-Walker (FRW) metric (with usual notations):

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2). \tag{10}$$

Since φ does not depend on spatial coordinates the Euler-Lagrange equations reduce simply to

$$\ddot{\varphi}(L_w + 2(\dot{\varphi})^2 L_{ww}) = -\frac{1}{2}\frac{\delta V}{\delta\varphi},$$

where a dot means a derivative with respect to the time. At this point we remark that since $w = (\dot{\varphi})^2$ the above equation reads

$$\ddot{\varphi}\Psi = -\frac{1}{2}\frac{\delta V}{\delta\varphi}. \tag{11}$$

Now, the effective invariant length element reads

$$ds^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = dt^2 - \frac{1}{L_w}(dr^2 + r^2 d\Omega^2). \tag{12}$$

Note that $\Psi = 1$, that is $2L_{ww} = 1 - L_w$, leads to $\hat{g}_{tt} = 1$ [14]. Let us set the expansion factor on the effective FRW geometry to an inflationary form: $a(t) = e^{Ht}$, H being a reel positive parameter. For that choice the equation

$$a(t)^2 = \frac{1}{L_w}$$

leads to

$$\sqrt{w} = \frac{\lambda}{e^{-2Ht} - 1}. \tag{13}$$

Since $\sqrt{w}$ is positive λ must be negative. Assuming $\dot{\varphi} \le 0$ (calculations for $\dot{\varphi} \ge 0$ are analogous), the above equation can be integrated to

$$\varphi = \frac{\lambda}{2H}\ln(e^{2Ht} - 1) + K, \tag{14}$$

where K is a constant, which we set equal to zero. Solving (14) for t allows one to integrate (11) to obtain precisely the form exhibited in Eq. (9) of the potential. In other words, observation of the effective geometry $\hat{g}_{\mu\nu}$ would lead us to believe, erroneously, that we live in a de Sitter geometry. Although we are dealing here with a toy model, a similar situation can occur for other nonlinear theories, like nonlinear electrodynamics.

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