Cosmic spinning string and causal protecting capsules

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A method by which a geometry with causality violation can be taken as a part of a globally causal space-time model is presented. This procedure is applied to a pair of extensions to Gödel's space-time: a "Godel-generalized" and a stringlike solution. The latter is shown to be an intermediate region between Gödel and deformed Minkowski geometries. Some conclusions and general ideas concerning structures with closed timelike curves are also presented.

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I. INTRODUCTION

In the last years great interest has been aroused concerning the study of metrical properties associated with infinite strings. It has been shown that the exterior geometry of the string is *conical,* associated with a *deficit* angle which depends uniquely on the linear density of the string [2–5]. Lately, in order to generate a more realistic situation, the case of a nonsingular string (constituting a cylinder of radius r_0 and energy density $\rho>0$, with a pressure along the z axis giving $\rho+p_z=0$) has been examined [6,7].

The natural evolution of such a study led to the introduction of rotation, generating spinning strings. A new qualitative feature then arises: the question of causality violations. Recently some physicists [8–14] dared to examine the difficulties to reconciliate standard ideas concerning causal problems in spaces admitting the occurrence of *closed timelike curves* (CTC's). This kind of curve appeared for the first time in the framework of general relativity through Gödel's cosmological solution [1]. Although there have been some comments [15] to the extent that Gödel's is an "artificial geometry," it is notwithstanding the true paradigm for space-times presenting CTC's and as such cannot be ignored. Nobody doubts that we do not live in a Gödel universe, but this does not preclude the interest in this special solution. In addition, as we shall show in this paper, Gödel's geometry can be enclosed in a compact region which possesses a well-behaved exterior extension. In other words, the theory does not forbid that Gödel's geometry exists in a limited bounded region encircled by a membrane (or a capsule) which maintains the required condition of global causality.

There are two most widely used attitudes concerning the above-mentioned causality problems: (i) reject all geometries presenting such features as physically undesirable; (ii) keep some form which preserves the most of traditional physics, e.g., the Cauchy initial value problem [10,11,16]. Some authors [17] embarked upon a program trying to connect theorems relating CTC's to physically forbidden situations, e.g., the violation of the weak energy condition, in a very similar way as was done for the singularity theorems in the 1960s.

This was criticized by Jensen and Soleng [7], who exhibited a solution in which the existence or nonexistence of CTC's was *not* related to the energy conditions. In the same framework, we shall show here that it is possible to exhibit a theoretical model, independent of the energy conditions, which may or may not present CTC's. At this point we will be able to introduce the idea of *capsules of causal protection.*

II. SYNOPSIS

In this article we exhibit a method by which any geometry admitting CTC's can generate a well-behaved global causal structure. In our case, this method will be applied for a particular example, namely, the Gödel geometry. In the next section we review briefly the geometry of spinning strings and the appearance of causality problems. In Sec. IV we examine a deformed Gödel universe, consisting of an extension of this geometry to a "Gödel-generalized" solution. Section V deals with the case of a second deformed Gödel universe, in which the vorticity changes with the radial coordinate. This allows us to achieve a connection between Gödel and deformed Minkowski geometries through an intermediate stringlike solution. We end with Sec. VI, in which some conclusions and general ideas concerning CTC's are presented.

III. THE DEFORMED VACUUM

The exterior metric of a spinning string can be written in the cylindrical coordinate system (t,r,φ,z) in the form

$$ds^2=a^2[dt^2-dr^2+2h(r)d\varphi\, dt+g(r)d\varphi^2-dz^2] \ . \tag{1}$$

In the absence of any energy content the functions h and g are given by

$$\begin{aligned} h(r)&=4GJ=\text{const} \ ,\\ g(r)&=(4GJ)^2-(1-4GM)^2r^2 \ . \end{aligned} \tag{2}$$

This geometry is locally flat but has global gravitational effects which have been examined by many authors (see, for instance, Refs. [2,3,6,7]). We will concentrate here on its causal properties. In order to clarify this let us perform a change of coordinates to a Gaussian system. A very particular choice made by Deser, Jackiw, and 't Hooft [18] and Mazur [19] yields a new time $T=t+4GJ\varphi$. This generates causal difficulties due to the cyclic character of φ.

One should wonder if this unusual choice, by imposing a cyclic condition on time, should not be responsible for such a strange causal behavior. That this is not the case can be shown by a simple inspection of all possible Gaussian coordinate systems for this geometry [20]. To provide for such a class of special coordinates one has to solve Jacobi's equation

$$g^{\mu\nu}\frac{\partial T}{\partial x^{\mu}}\frac{\partial T}{\partial x^{\nu}}=1$$

for the geometry (1). It then follows that the Gaussian tim T is given by the ansatz

$$T=\lambda_1 t+\lambda_2\varphi+\lambda_3 z+F(r) \tag{3}$$

in which

$$F(r)=\sqrt{\mu^2r^2-\nu^2}+\nu\arcsin(\nu/\mu r)$$

and the constants μ and ν are given in terms of the parameters λ_i by the expressions

$$\mu^2=\lambda_1^2-1\ ,$$

$$\nu=\frac{1}{\alpha}(4GJ\lambda_1-\lambda_2)\ .$$

Just for completeness, let us exhibit the remaining spacial coordinates ξ, η, and z':

$$\xi=t+\frac{\lambda_1}{\mu^2}\sqrt{\mu^2r^2-\nu^2}+\frac{4GJ}{\alpha}\arcsin\left[\frac{\nu}{\mu r}\right]\ ,$$

$$\eta=\varphi-\frac{1}{\alpha}\arcsin\left[\frac{\nu}{\mu r}\right]\ ,$$

$$z'=z\ .$$

The geometry then takes the Gaussian form

$$ds^2=dT^2-\mu^2d\xi^2-[16G^2J^2\mu^2+(1-4GM)^2r^2]d\eta^2 -8GJ\mu^2d\xi\,d\eta-dz'^2 \tag{4}$$

in which $r=-\mu T+\mu\lambda\xi+4GJ\mu\lambda\eta$. We note that the determinant of $g_{\mu\nu}$, given by

$$\det g_{\mu\nu}=-\mu^4(1-4GM)^2(-T+\lambda\xi+4GJ\lambda\eta)^2\ ,$$

becomes singular at the hypersurface $T=\lambda(\xi+4GJ\eta)$.

It is worth examining two particular values of the parameter ν.

Case (i): $\nu=0$

A global Cauchy surface is provided by the equation $T=$const. The new time is defined by means of a family of geodesics which intersects a global spacelike hypersurface Σ. In this case the parameter λ_2 cannot be made zero, which then results in T being cyclic: $T=\lambda(t+4GJ\varphi)$. For a matter of continuity the values T_0 and $T_0+8GJ\pi\lambda\varphi$ must be identified.

Case (ii): $\nu\neq 0$ *with* $\lambda_2=0$

In this case the Gaussian coordinate system has a *hole.* The parameters μ and ν do not vanish but the Gaussian system is defined only for

$$r>r_g\equiv\nu/\mu\ .$$

The region where this system is not defined contains closed timelike curves (CTC's), once $g(r)<0$ for $r<r_c\equiv 4GJ/(1-4M)$. We remark that $g(r)<0$ constitutes the condition that the curve with constants t, r, and z is closed.

In both cases presented above we face a causality problem. This is probably a consequence of the idealized configuration of the structureless string that one deals with in these cases. Jensen and co-worker [6,7] support this argument. They exhibit an internal solution concerning the spinning string adapted to an exterior Minkowskian geometry in such a way that the radius of the string stands beyond the previous acausal domain, thus inhibiting any causal deficiencies to appear. The source of this internal solution is a fluid endowed with anisotropic pressure $\pi_{\mu\nu}$ and a nonvanishing heat flux q_μ in the φ direction. The geometry has the form presented in (1) in which functions $h(r)$ and $\Delta(r)\equiv\sqrt{h^2-g}$ are given by

$$h_{\rm I}(r)=(r-r_S)\cos\lambda r-\frac{1}{\lambda}\sin\lambda r+r_S\ ,$$
$$\Delta_{\rm I}(r)=b\sin\lambda r\ , \tag{5}$$

where the index I stands for the internal geometry and r_S is the string radius; likewise the index II will stand for the external geometry. Note that λ is related to the matter content of the solution.

This solution must be connected to (2) obeying the Darmois-Lichnerowicz (DL) conditions [21]. Choosing the hypersurface Σ, the boundary between the two regions, to be given by the radius $r=r_S$, the DL conditions reduce to the continuity of the functions h and Δ and their respective derivatives through Σ.

Denoting $[h]\equiv h_{\rm I}-h_{\rm II}$ as the discontinuity through the hypersurface Σ, the DL conditions are

$$[h]=[\Delta]=[h']=[\Delta']=0 \tag{6}$$

in which a prime indicates derivatives with respect to the radial coordinate. Applying condition (6) in the above solution it follows that its free parameters (J, M, λ, r_S, and b) must satisfy

$$4GJ=r_S-\frac{1}{\lambda}\sin\lambda r_S\ , \tag{7}$$

$$(1-4GM)=b\lambda\cos\lambda r_S\ , \tag{8}$$

$$b\sin\lambda r_S=(1-4GM)r_S-4GJ\ . \tag{9}$$

We note that only two parameters (M and J) are completely independent physical quantities. That is, by a convenient choice of M and J one can control the appearance of CTC's in the different models. Then, by filling in the interior region with a convenient matter configuration, respecting the previous symmetry, one can drastically modify the causality properties of the geometry. This procedure will be generalized further on and, as we shall see in a subsequent section, it can support a conjecture regarding the absence of causality violations in our Universe.

IV. THE DEFORMED GÖDEL'S UNIVERSE I

Let us come back to the form (1) of the geometry. Choose a tetrad frame $e_A^{(\alpha)}$ (in which A is a tetrad index) given by

$$e_0^{(0)}=e_1^{(1)}=e_3^{(3)}=a \ ,$$
$$e_2^{(0)}=-h/a\Delta \ , \tag{10}$$
$$e_2^{(2)}=1/a\Delta \ .$$

In this frame the only nonidentically zero Ricci components are

$$R_{00}=-\frac{1}{2a^2}\left[\frac{h'}{\Delta}\right]^2 , \tag{11}$$

$$R_{11}=\frac{1}{a^2}\left[\frac{\Delta''}{\Delta}-\frac{1}{2}\left[\frac{h'}{\Delta}\right]^2\right] , \tag{12}$$

$$R_{22}=R_{11} \ , \tag{13}$$

$$R_{02}=-\frac{1}{2a^2}\left[\frac{h'}{\Delta}\right] . \tag{14}$$

The geodesic congruence generated by a set of comoving observers $V^A=\delta_0^A$ has no shear or expansion but has a non-null vorticity vector

$$\omega^A=\left[0,0,0,-\frac{1}{2a}\frac{h'}{\Delta}\right] .$$

We define the quantity

$$\omega(r)=-\frac{1}{2}\frac{h'}{\Delta}$$

as the vorticity. In the Minkowski geometry we have $\omega(r)=0$. We now consider the case in which $\omega(r)$ is a constant ω_0 different from zero. This was precisely the case studied by Gödel in his remarkable 1949 paper [1]. From this condition and using (14) it follows that the source of this geometry has no heat flux. If the pressure vanishes, the energy density ρ, the vorticity ω_0, and the cosmological constant Λ are related, through Einstein's equations, by

$$\rho=2\omega_0^2=-2\Lambda \tag{15}$$

yielding Gödel's solution

$$h(r)=\sqrt{2}\sinh^2 r \ , \tag{16}$$

$$\Delta(r)=\sinh r\cosh r \ . \tag{17}$$

The analysis of the whole set of timelike and null geodesics in this geometry shows that the vorticity provides for a strong attractive power that produces a confinement of all free particles within a *critical radius* $r_C=\text{arcsinh}1$. This confinement is responsible for the impossibility of constructing a unique Gaussian system of coordinates which allows the existence of CTC's beyond r_C (see, for instance, Ref. [20]).

In fact, Gödel's geometry was the first cosmological model known to admit CTC's and became a paradigm for causality violations in Gravitational theory.

As a general method of attack to establish a framework to inhibit the occurrence of CTC's, one could try to make use of the experience gained in the previous section and deform this geometry, by destroying its homogeneity, notwithstanding preserving its axial symmetry.

Our problem can thus be defined as follows: to find a new solution of Einstein's equations for a geometry with the form given by (1), which can be joined to an interior Gödel's geometry through a hypersurface Σ defined by the equation $r=r_\Sigma=\text{const}$. If this can be done for an arbitrary value of r_Σ, then we could choose it as less than the confining region ($r_\Sigma<r_C$) to inhibit the appearance of closed timelike curves, in the same way as was done in Sec. III.

We thus set for the stress-energy tensor the form

$$T_{\mu\nu}=\rho V_\mu V_\nu-ph_{\mu\nu}+\pi_{\mu\nu}+q_{(\mu}V_{\nu)} \ . \tag{18}$$

It then follows that the quantity $\omega(r)$, defined earlier in this section, is a constant (i.e., no heat flux).

As a direct consequence of the axial symmetry of the metric (1), the only nonzero components of the anisotropic pressure tensor satisfy the relation

$$\pi_{11}=\pi_{22}=-\tfrac{1}{2}\pi_{33}\equiv\pi_0 \ . \tag{19}$$

We find the solution of Einstein's equations as

$$\Delta(r)=\frac{1}{\lambda}\cosh 2r_\Sigma\sinh u+\tfrac{1}{2}\sinh 2r_\Sigma\cosh u \ , \tag{20}$$

$$h(r)=\frac{2\sqrt{2}}{\lambda^2}\cosh 2r_\Sigma(\cosh u-1)+\frac{\sqrt{2}}{\lambda}\sinh 2r_\Sigma\sinh u+\sqrt{2}\sinh^2 r_\Sigma \ , \tag{21}$$

with $u\equiv\lambda(r-r_\Sigma)$, in which λ is a constant related to the matter terms by

$$\lambda^2=a^2(2p-\pi_0-2\Lambda) \tag{22}$$

and r_Σ is a positive constant. The isotropic (p) and anisotropic (π_0) pressure are given by

$$\pi_0=\tfrac{1}{2}(\rho+p-2\omega^2) \ , \tag{23}$$

$$p=\tfrac{2}{3}(\Lambda+2\omega^2-\tfrac{1}{2}\rho) \ . \tag{24}$$

Let us emphasize that the weak and the strong energy conditions can be satisfied for convenient choices of the free parameters. We can now join this solution to Gödel's geometry through the hypersurface $r=r_\Sigma$, satisfying DL conditions [Eq. (6)].

The final geometrical configuration is thus the following: for $0<r<r_\Sigma$ the geometry is provided by Gödel; for $r>r_\Sigma$ the geometry has the same cylindrical symmetry but the metrical coefficients $h(r)$ and $\Delta(r)$ are given by (20) and (21). This exterior solution can be considered as

"Gödel generalized," since it was proven that is reduces to Gödel for the special case $\lambda=2$.

Now we can consider the causality properties of this structure. Choosing the connection point to be given by $r_\Sigma=\delta \operatorname{arcsinh} 1$ for $\frac{1}{2}<\delta<1$, it follows that CTC's are completely forbidden if the anisotropic pressure and the cosmological constant are bounded, i.e.,

$$-\frac{4}{3a^2(\sqrt{2}-1)}<\pi_0<-\frac{4}{3a^2}\ ,$$
$$\Lambda<\frac{2}{a^2}+3\pi_0\ . \tag{25}$$

There are also two other matter configurations which provide valid solutions, but we will not consider them here.

V. THE DEFORMED GØDEL'S UNIVERSE II: THE NONCONSTANT VORTICITY

In this section we will combine the results of what we have learned above in order to generate a more complex structure than the previous one. We have studied a model which joined two different solutions of Einstein's equations: the interior (Gödel) and a new exterior solution with the same constant vorticity for all values of the radial coordinate r.

As we remarked before, our goal is to join Gödel's solution to a deformed Minkowski geometry (i.e., one with a topological defect—an angular *deficit*). Both solutions possess constant vorticity, but their values are not the same. To achieve a valid model, we must look for an intermediate solution, with vorticity $\omega(r)$, which varies from ω_0 (Gödel's value) to zero (deformed Minkowski's value).

This intermediate solution is found to be a general case of the spinning string, as presented by Jensen and Soleng [6,7]. The solution presented by these authors differs from ours in the sense that it tends to Minkowski geometry for $r=0$, whereas our solution must tend necessarily to Gödel's geometry for the interior region (defined by a given width).

In order to make our calculations more systematic, we will thus consider a model containing three regions: region I, Gödel's solution; region II, a spinning string solution; region III, a deformed Minkowski solution. As can be verified, regions I and II are described by solutions with nonzero cosmological constant, whereas region III is not. However, this can be justified by the fact that the characterization of a given *source* of the gravitational field is not unique (this has been shown a number of times in the literature; in particular, we can quote Ref. [22], in which it was shown that the source of a Gödel-like geometry can be equivalently described either as a perfect fluid, with a non-null Λ, or as a perfect fluid plus an external electromagnetic field).

As we are interested in solutions with nonconstant vorticity, Eq. (18) implies that the source of the geometry must have a nonzero heat flux. This can be seen combining Einstein's equation for R_{02} with (18), which gives

$$\left[\frac{h'}{2a^2\Delta}\right]'\equiv\omega'(r)=q_\varphi\ . \tag{26}$$

Let us limit ourselves here to the special case in which $q_\varphi=$const. We will see that this condition is sufficient for our purposes. Now we are able to perform a simple integration in (26), obtaining

$$\frac{h'}{2a^2\Delta}=q_\varphi(r+\xi)\ , \tag{27}$$

where ξ is an arbitrary integration constant, to be determined soon. We will also redefine the parameter λ as

$$\lambda^2\equiv a^2(-2p+\pi_0+2\Lambda)\ , \tag{28}$$

maintaining it positive, as in the previous section. In this case, Eq. (14) gives

$$\Delta''+\lambda^2\Delta=0\ , \tag{29}$$

which is easily solved for $\lambda\equiv$const.

The constant ξ can be obtained if we impose that the solution for region II is joined exteriorly (for a radius r_S) to deformed Minkowski geometry, given by [6,7]

$$h_{\rm III}(r)=4GJ\ , \tag{30}$$

$$\Delta_{\rm III}(r)=(1-4GM)(r+r_0)\ , \tag{31}$$

where J is the string's angular momentum per length, M is the string's mass per length, and r_0 is an arbitrary constant. Imposing DL conditions (continuity for $r=r_S$) we obtain

$$\omega(r_S)=0\Longrightarrow\xi=-r_S\ . \tag{32}$$

Substituting (32) in (27) and solving Einstein's equations we find the desired spinning string solution for region II as

$$\Delta_{\rm II}(r)=A\sin\lambda r+B\cos\lambda r\ , \tag{33}$$

$$h_{\rm II}(r)=\frac{2a^2q_\varphi A}{\lambda}\left[\frac{1}{\lambda}\sin\lambda r+(r_S-r)\cos\lambda r\right]+\frac{2a^2q_\varphi B}{\lambda}\left[\frac{1}{\lambda}\cos\lambda r-(r_S-r)\sin\lambda r\right]+\alpha\ . \tag{34}$$

Einstein's equations and (28) give also the following result for the energy density:

$$\rho=3\pi_0-\Lambda+a^2q_\varphi^2(r_S-r)^2 \tag{35}$$

$$=\left[\frac{\lambda}{a}\right]^2-\Lambda+3a^2q_\varphi^2(r_S-r)^2\ . \tag{36}$$

Now we apply DL junction conditions for the complete mode, given by region I ($0\le r\le r_\Sigma$), Gödel's solution, Eqs. (16) and (17), region II ($r_\Sigma\le r\le r_S$), the spinning string generalized solution, Eqs. (33) and (34), and region III ($r\ge r_S$), the deformed Minkowski solution, Eq. (31). Note, in order to obtain an analytically joined model, these conditions must be satisfied for two hypersurfaces:

$r=r_\Sigma$ and $r=r_S$. Consequently, this gives us two sets of equations of the kind of Eq. (6), which must be simultaneously valid. Therefore, these equations provide us with the means to obtain the integration constants A, B, α, and r_0 and J, M, and q_φ, related to the matter which generates the solution for region II, in terms of the parameters λ, r_Σ, and r_S.

Thus applying DL conditions results in the relations

$$A=\frac{1}{\lambda}[\lambda(\sinh r_\Sigma \cosh r_\Sigma)+(2\sinh^2 r_\Sigma+1)\cos\lambda r_\Sigma] , \quad (37)$$

$$B=\frac{1}{\lambda}[\lambda(\sinh r_\Sigma \cosh r_\Sigma)-(2\sinh^2 r_\Sigma+1)\sin\lambda r_\Sigma] , \quad (38)$$

$$\alpha=\frac{\sqrt{2}}{\lambda^2(r_S-r_\Sigma)}[2\sinh r_\Sigma \cosh r_\Sigma +2(r_s-r_\Sigma)(2\sinh^2 r_\Sigma+1)] +\sqrt{2}\sinh^2 r_\Sigma , \quad (39)$$

$$r_0=\frac{\lambda(\sinh r_\Sigma \cosh r_\Sigma)\cos u+(2\sinh^2 r_\Sigma+1)\sin u}{(2\sinh^2 r_\Sigma+1)\cos u-\lambda(\sinh r_\Sigma \cosh r_\Sigma)\sin u}-r_S , \quad (40)$$

$$q_\varphi=-\frac{\sqrt{2}}{a^2(r_S-r_\Sigma)} , \quad (41)$$

$$M=\frac{1}{4G}[1-(2\sinh^2 r_\Sigma+1)\cos u] +\frac{\lambda}{4G}\sinh r_\Sigma \cosh r_\Sigma \sin u , \quad (42)$$

$$J=\frac{\sqrt{2}}{2G\lambda^2(r_S-r_\Sigma)}(\sinh r_\Sigma \cosh r_\Sigma)(1-\cos u) +\frac{\sqrt{2}}{2G\lambda^3(r_S-r_\Sigma)}(2\sinh^2 r_\Sigma+1)[\lambda(r_S-r_\Sigma)-\sin u] +\frac{\sqrt{2}}{4G}\lambda^3(r_S-r_\Sigma)\sinh^2 r_\Sigma , \quad (43)$$

where u stands again for $\lambda(r_S-r_\Sigma)$.

By a convenient choice of the three parameters λ, r_Σ, and r_S, one can obtain models that do not violate causality for any value of the coordinate r. All that is required is that the function $g(r)$ satisfy the inequality

$$g(r)\equiv h^2-\Delta^2<0$$

for each region simultaneously. We also note that this model successfully joins a region with constant vorticity (Gödel) to another with zero vorticity (deformed Minkowski) through an intermediate region (generalized spinning string) which presents variable vorticity (in terms of the coordinated r). The quantity ω is given, for each region, by region I, $\omega\equiv\omega_0=2/a^2$, region II,

$$\omega\equiv\omega(r)=\frac{2(r_S-r)^2}{a^2(r_S-r_\Sigma)^2} ,$$

and region III, $\omega=0$. It is easy to verify that $\omega(r)$ is continuous both for $r=r_\Sigma$ and for $r=r_S$, as it should.

The above structure provides a way to suspend the occurrence of causality violations in Gödel's geometry, by the presence of a *protecting capsule.*

VI. CONCLUSIONS

Nowadays causality is still an open question. Concerning this problem there are three typical attitudes: (i) the laws of physics forbid the appearance of closed timelike curves (Hawking) [17]; (ii) the laws of physics allow for CTC's and nature exhibits them (Thorne and co-workers) [16,23]; (iii) the laws of physics allow the appearance of CTC's, but nature organizes itself in such a way as to hide them. In this paper we have opted for attitude (iii) by exhibiting a specific case in which Nature can make CTC's inaccessible. We argue that the best way to achieve this result is to examine Gödel's geometry, once it is the true paradigm of solutions which admit CTC's, although this method can be generalized for other kinds of metric. A complex structure is produced, consisting of solutions of Einstein's equations for different continuous regions, in such a way that only part of Gödel's geometry, bounded by its critical radius (which delimits the separation between a well-behaved region and the one in which CTC's can occur) is considered. This region possesses an analytical continuation (i.e., one which satisfies the standard Darmois-Lichnerowicz junction conditions) to a topologically deformed Minkowski solution, with a generalized spinning string solution in between them. This structure leads us to a further conjecture: *all would-be CTC's regions of a given geometry are occupied by other geometries which do not allow closed timelike curves.* These *causal protecting capsules* would thus prevent causality violation from occurring by enclosing the causal region of the basic geometry inside another geometry, without causality violation.

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