Extended Born-Infeld theory and the bouncing magnetic universe

M. Novello,* J. M. Salim, and Aline N. Araújo

Institute of Cosmology, Relativity and Astrophysics (ICRA/CBPF), Rua Dr. Xavier Sigaud, 150, CEP 22290-180, Rio de Janeiro, Brazil

(Received 9 December 2011; published 25 January 2012)

We show that a generalized Born-Infeld electrodynamics responsible for regular configurations of the static field of a charged particle produces a nonsingular universe that contains a bouncing. This means that the Universe has a previous collapsing phase, attains a minimum value for its scale factor and then enters into an expanding phase. We exhibit such a scenario in the case of an average pure magnetic universe. At its infinity past as well as at its infinite future the distribution of the energy content of the magnetic fluid displays the form of a cosmological constant. Thus such a configuration is an intermediary between asymptotic vacuum states. In other words, this magnetic universe evolves from vacuum to vacuum.

DOI: 10.1103/PhysRevD.85.023528 PACS numbers: 98.80.Cq

I. INTRODUCTION

In their original seminal paper in 1934, Born and Infeld [1] argued that Maxwell linear electrodynamics must be changed in order to give origin to a well-behaved classical field theory that does not violate "... the principle of finiteness which postulates that a satisfactory theory should avoid letting physical quantities become infinite." In their analysis, emphasis was given to the static electric field with spherical symmetry corresponding to a charged body generating a regular electric field configuration, showing no singular behavior. In other words, the field has an absolute maximum.

Their analysis was limited to the electromagnetic effects. The other long-range field *gravity* was not involved in their work. The reason is simple to understand: the extreme weakness of gravitational coupling makes unnecessary in most cases the introduction of processes depending on gravity. However, there is one important configuration in which gravity cannot be forgotten, and this is cosmology. If we introduce gravitational effects controlled by the theory of general relativity, a drawback of Born-Infeld proposal appears: as in the case of the linear Maxwell theory, there is no place for a regular cosmological scenario for the combined effects of gravity and electrodynamics. Such a disadvantage remained throughout all of these years. It is true that in the last decade there has been an increase in the interest of cosmological effects induced by nonlinear electrodynamics (NLED) [2,3]. The main reason for this is related to the drastic modification that NLED provokes in the behavior of the cosmological geometry with respect to two of the most important questions of standard cosmology, that is, the initial singularity and the acceleration of the scale factor. However, these proposals have not tackled the combined property of such a regular cosmological framework with the finiteness of the field in the vicinity of a charged particle. The purpose of the present work is to provide a theory in which it is possible to achieve regular behavior of the electromagnetic field in both situations, that is, in the neighborhood of a charged particle and in the cosmological framework.

A. Magnetic universe

Modern cosmology states that the Universe is spatially homogeneous and isotropic and contains mainly photons, matter concentrated in compact structures (usually taken as a simple perfect fluid configuration) and some extra form of energy that still is not known. From these ingredients cosmological models have been produced that exhibits a singular behavior.

To circumvent this unpleasant cosmological origin many modifications of the equations of Einstein theory have been proposed. However, it is very possible that the kernel of the difficulty does not belong to the gravity description but instead concerns the other long-range field, as we will show. Indeed, in the present paper gravity will be described by general relativity and electrodynamics by a nonlinear theory that contains some advantages over the Maxwell linear form in the regime of a very strong field along the same lines as the Born-Infeld proposal [4] and that will become explicit in the present work.

The general form for the dynamics of the electromagnetic field, compatible with covariance and gauge conservation principles [5] reduces to $L = L(F)$, where $F \equiv F^{\mu\nu}F_{\mu\nu}$. Once the main novelty of our modification of Born-Infeld electrodynamics concerns cosmology, let us anticipate here some of these new properties.

The arguments presented in [6] make it worth considering that only the averaged magnetic field survives in a spatially homogeneous and isotropic geometry. Such configuration of pure averaged magnetic field combined with the dynamic equations of general relativity received the generic name of the magnetic universe [7]. The most remarkable property of a magnetic universe configuration is the fact that from the energy conservation law it follows that the dependence on time of the averaged magnetic field $B(t)$ is the same irrespective of the specific form of the

*ICRANet Cesare Lattes Professor

Lagrangian. This property allows us to obtain the dependence of the magnetic field on the scale factor $a(t)$, without knowing the particular form of the Lagrangian $L(F)$. Indeed, as we will show later on, from the energy-momentum conservation law it follows that $B = B_0 a^{-2}$.

It is a well-known fact that the standard cosmological model unavoidably leads to a singular behavior of the curvature invariants in what has been termed the big bang. This is a highly distressing state of affairs, because in the presence of a singularity we are obliged to abandon the rational description of Nature. It is possible that a complete quantum cosmology could describe the state of affairs in a very different and more complete way. For the time being, while such complete quantum theory is not yet known, one should attempt to explore alternatives that are allowed and that provide some sort of phenomenological consequences of a more profound theory. The combined effects of Maxwell linear electrodynamics with the equations of general relativity in a spatially homogeneous and isotropic metric implies the unavoidable and disturbing presence of a singularity. Such an undesirable property remains if the nonlinear effects induced by the Born-Infeld theory are taken into account. It is precisely to extend the regular character introduced in their formulation of an electric field of a charged body into the cosmological scenario that led us to propose the present modified version of Born-Infeld theory.

Scenarios that avoid an initial singularity have been intensely studied over the years (see Novello-Santiago [8] for a complete review). In the present article we limit ourselves to the simplest model generated by the coupling of gravity and an electromagnetic field. Thus, we present a cosmological scenario controlled by the energy density ρ and the pressure p of an average magnetic field (see the next section for the meaning of this average). The main features of this scenario can thus be synthesized by the following few steps:

(i) The Universe is dominated by an average magnetic field;
(ii) At the infinite past the energy distribution representing the asymptotic regime of the magnetic field is equivalent to a cosmological constant;
(iii) This initial configuration is not stable: the Universe collapses;
(iv) The Universe stops its contraction when arriving at the minimum value of the scale factor $a_{\min} = a_b$;
(v) In this state the density of energy has a minimum;
(vi) Then it starts a phase of expansion: the volume starts to increase and continue being always accelerated;
(vii) This universe ends in a similar configuration for the magnetic fluid as it was at its beginning, that is identical to a perfect fluid with the equation of state of the vacuum $\varrho + p = 0$, and the volume goes to infinity.

There are some interesting features in this scenario. Let us anticipate two of them relating to the property that near the bouncing, in the collapsing phase there appears an unusual behavior of the fluid: although the volume is diminishing (in the collapsing phase) the density of energy ϱ decreases. After passing the minimum value for the scale factor, ϱ starts to increase although the volume is increasing. This behavior is due to the high negative value of the pressure. After passing the point in which the density of energy has a maximum, the behavior of ϱ changes and become the expected one, that is, increasing of the volume is accompanied with a diminishing of the density ϱ. Such behavior occurs also in other scenarios (see, for instance [9]).

This scenario is akin to the idea of Lemaître primordial "atom." However, contrary to his suggestion, in the present model at its initial stage the Universe is very big. This primordial configuration is not stable and cannot remain as such: it starts to collapse due to the negative pressure. We shall prove that this collapses stops at a certain point and the universe starts to expand, avoiding the undesirable passage through a singularity contained in Friedmann models. Thus, this scenario describes what happened in the neighborhood of the maximum condensate point: a bounce.

The article is organized as follows. In Sec. II we describe our theory of nonlinear electrodynamics and compare its properties with the Maxwell linear model and Born-Infeld. Section III provides a short review of the nonlinearity of the theory that allows its description in terms of the field inside matter defining the effective dielectric and susceptibility parameter in terms of functions of the field that depends on its Lagrangian. In Sec. IV we review the Tolman process of average in order to reconcile the energy distribution of the electromagnetic field with a spatially isotropic geometry. Section V presents the notion of the magnetic universe and its generic features concerning the dynamics of electromagnetic field generated by a Lagrangian $L = L(F)$. Sections VI and VII deal with the geometrical and dynamical properties of our model and a comparison with the previous results of the linear theory and Born-Infeld proposal. We end with some conclusions and future perspectives. In Appendix A we present the compatibility of our Lagrangian with the standard Coulomb law.

Finally, let us note that we limited our considerations here to the case of magnetic field neglecting the matter and energy of other forms. This was made in order to simplify our scenario and its analysis and to make clear the main properties of our new theory. We made only a few comments concerning the modification in the dynamics of the cosmological metric when incoherent matter (no pressure) is taken into account, leaving the complete analysis of the effects introduced by other forms of matter/energy for a more extended (and more realistic) version to a subsequent paper.

II. THE THEORY

The method proposed by Born and Infeld to limit the maximum possible value of the electric field is already contained in the action principle that drives its dynamics. This is achieved by setting the form of the Lagrangian as

$$L = -\gamma^2 U^\sigma, \tag{1}$$

where

$$U \equiv 1 + \beta F$$

and $\sigma \equiv (2n+1)/2$, n is any natural number.

Their analysis was made only for the case $n = 0$, and we will restrict our investigation here only to their choice. In order that this theory reduces to the linear Maxwell limit for very small fields (comparing with γ^2) they set

$$\beta = \frac{1}{2\gamma^2}.$$

This dynamics was constructed under the hypothesis that there must exists a maximum possible value of the electric field and that the theory must contain explicitly such a constraint. Nothing similar is demanded for the magnetic sector. For the situations dealing with charged bodies, this restriction is sufficient to avoid divergencies. However, in the cosmological framework, as in the case of magnetic universe that we analyze here, this is not enough as we will show in the next section. This led us to generalize the Born-Infeld dynamics in such a way that the new theory allows for an absolute maximum for the field in any circumstance, or in other words, to limit the invariant $F = -2E^2 + 2B^2$ to the bounds $X_- < F < X_+$. To achieve this condition we set the Lagrangian (a scalar version of such extended Born-Infeld type of theory was presented in [4])

$$L = -\gamma^2 W^{1/2}, \tag{2}$$

where

$$W = 1 + \beta F - \alpha^2 F^2. \tag{3}$$

The Born-Infeld theory is the particular case in which $\alpha = 0$. In order that this theory reduces to the linear Maxwell limit for very small fields (comparing with γ^2) we set

$$\beta = \frac{1}{2\gamma^2}. \tag{4}$$

Note that the next term of the series yields the quadratic Euler-Heisenberg correction. From the form of this Lagrangian it follows that the field has two extremum provided by the values X_- and X_+ that limit the electric and the magnetic fields:

$$X_- = \frac{1 - \sqrt{1 + 16\alpha^2\gamma^4}}{4\alpha^2\gamma^2}$$

and

$$X_+ = \frac{1 + \sqrt{1 + 16\alpha^2\gamma^4}}{4\alpha^2\gamma^2}.$$

III. NONLINEAR THEORY AS A PLASMA

It has been pointed out [10] that nonlinear electrodynamics is a useful tool to present in an equivalent formal way the effects of charged bodies in a plasma configuration. Let us present very briefly an overview of such a concept. We start by noting that the equation of motion of a theory described by a Lagrangian $L(F)$ is given by

$$(L_F F^{\mu\nu})_{;\nu} = 0, \tag{5}$$

where $L_F = dL/dF$. This expression can be rewritten in a form that is worth analyzing in the cosmological framework, that is,

$$F^{\mu\nu}_{;\nu} = J^\mu, \tag{6}$$

where

$$J^\mu = -\frac{L_{FF}}{L_F} F^{\mu\nu} F_{,\nu}.$$

Using our Lagrangian (2) it follows that

$$\frac{L_{FF}}{L_F} = -\frac{1}{4\gamma^2 W} \frac{(1 + 16\alpha^2\gamma^4)}{(1 - 4\alpha^2\gamma^2 F)}.$$

In the case of a spatially homogeneous universe F is function only of the cosmological time. This means that the current reduces to the form

$$J^\mu = \sigma E^\mu.$$

In the asymptotic limit where the field is very weak (we shall see that in the case of a spatially homogeneous and isotropic geometry such situation occurs at the infinite past) σ is a constant which in our theory is given by

$$\sigma \approx \frac{1 + 16\alpha^2\gamma^4}{4\gamma^2}.$$

Using this result into Eq. (6) it follows that the nonlinearities mimic a plasma even in the case in which there are no currents at all. The effective current described by σE^μ drives the field as if there was a real plasma. We shall use this result into the Tolman average procedure in the next section.

IV. THE AVERAGE PROCEDURE AND THE FLUID REPRESENTATION

The effects of a nonlinear electromagnetic theory in a cosmological setting have been studied in several articles [11]. In the standard cosmological scenario the metric structure of space-time is provided by the Friedmann-Lemaître (FL) geometry. For compatibility with the cosmological framework, that is, in order that an electromagnetic field can generates a homogeneous and isotropic geometry an average procedure must be used. There are distinct ways to define such an average. For the purpose of the present paper it is enough to suppose that for the electromagnetic field to act as a source for the FL model [12]

$$\bar{E}_i = 0, \qquad \bar{B}_i = 0, \qquad \overline{E_i B_j} = 0, \tag{7}$$

$$\overline{E_i E_j} = -\tfrac{1}{3}E^2 g_{ij}, \qquad \overline{B_i B_j} = -\tfrac{1}{3}B^2 g_{ij}, \tag{8}$$

where the symbol $\bar{X}$ means such an average value [13].

With these conditions, the energy-momentum tensor of the electromagnetic field associated to $L = L(F)$ can be written as that of a perfect fluid. Indeed, for a generic gauge-independent Lagrangian $L = L(F)$, written in terms of the invariant $F \equiv F_{\mu\nu}F^{\mu\nu}$, it follows that the associated energy-momentum tensor, defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta L \sqrt{-g}}{\delta g^{\mu\nu}}, \tag{9}$$

reduces to

$$T_{\mu\nu} = -4L_F F_\mu{}^\alpha F_{\alpha\nu} - L g_{\mu\nu}. \tag{10}$$

Then,

$$T_{\mu\nu} = (\rho + p)v_\mu v_\nu - p g_{\mu\nu}, \tag{11}$$

where

$$\rho = -L - 4L_F E^2, \qquad p = L - \tfrac{4}{3}(2B^2 - E^2)L_F. \tag{12}$$

V. MAGNETIC UNIVERSE

A particularly interesting case occurs when only the average of the magnetic part does not vanishes and we can set $E^2 = 0$. Such situation has been investigated in the cosmological framework yielding what has been called the *magnetic universe*. In the previous section we analyzed this and argued that it is a real possibility in the case of cosmology, since in the spatially homogeneous universe the nonlinearities can be interpreted in terms of a current that allows the electric field to become screened like in a charged plasma, while the magnetic field lines are frozen [6]. A remarkable feature of the magnetic universe comes from the fact that it can be associated to a perfect fluid. In spite of this fact, in [3] some attention was devoted to the mathematically interesting case in which $E^2 = \sigma^2 B^2 \neq 0$. We work with the standard form of the FL geometry in Gaussian coordinates provided by

$$ds^2 = dt^2 - a(t)^2 d\sigma^2. \tag{13}$$

We restrict our analysis to the case in which the curvature of the space sector is positive. The expansion factor θ defined as the divergence of the fluid velocity reduces, in the present case, to the derivative of logarithm of the scale factor

$$\theta \equiv v^\mu_{;\mu} = 3\frac{\dot{a}}{a}. \tag{14}$$

The conservation of the energy-momentum tensor projected in the direction of the comoving velocity $v^\mu = \delta^\mu_0$ yields

$$\dot{\rho} + (\rho + p)\theta = 0. \tag{15}$$

Substituting the values of the density of energy and pressure from the expressions above into the conservation law, it follows that

$$L_F\left[(B^2)^{\cdot} + 4B^2\frac{\dot{a}}{a}\right] = 0. \tag{16}$$

The important result that follows from this equation is that the dependence on the specific form of the Lagrangian appears as a multiplicative factor. This property shows that any Lagrangian $L(F)$ yields the same dependence of the field on the scale factor irrespective of the particular form of the Lagrangian. Indeed, Eq. (16) yields

$$B = B_0 a^{-2}. \tag{17}$$

This property implies, for instance, that if we develop the Lagrangian in a power series, it follows that for each power F^k it is possible to associate a specific fluid configuration with density of energy ρ_k and pressure p_k in such a way that the corresponding equation of state is given by

$$p_k = \left(\frac{4k}{3} - 1\right)\rho_k. \tag{18}$$

Using Lagrangian L given by (2) in the case of the magnetic universe yields for the density of energy and pressure given in Eq. (12):

$$\rho = \gamma^2 W^{1/2} \tag{19}$$

$$p = -\frac{\gamma^2 W^{-1/2}}{3}[3W - 2(\beta F - 2\alpha^2 F^2)], \tag{20}$$

where

$$F = 2\frac{B_0^2}{a^4}. \tag{21}$$

It is convenient to rewrite the pressure in terms of the density. We find

$$p = -\varrho + \frac{\gamma^2 F(1 - 4\alpha^2\gamma^2 F)}{3\varrho} \tag{22}$$

from which it follows that there exists a special value of the field that we will call F_c such that for values of the field greater than F_c then $\varrho + p$ is negative and for values of $F < F_c$ then $\varrho + p$ is positive.

Let us now turn to the analysis of the equations of general relativity which, due to the symmetry imposed on the metric structure reduces to three: a constraint given by

$$\varrho - \frac{3}{a^2} = \frac{\theta^2}{3} \tag{23}$$

and two dynamics that we examine next.

VI. DYNAMICAL SYSTEM

Besides the conservation of the energy density we must examine the evolution of the scale factor which is controlled by the Raychaudhuri equation of the expansion $\theta \equiv 3\dot{a}/a$, that is,

$$\dot{\theta} + \frac{\theta^2}{3} = -\frac{1}{2}(\varrho + 3p). \tag{24}$$

It is worth rewriting the system of equations of the magnetic universe in terms of an autonomous planar system. Instead of using variables (ϱ, θ) it is convenient to use (F, θ). We then write

$$\dot{F} = -\tfrac{4}{3}F\theta \tag{25}$$

$$\dot{\theta} = -\frac{\theta^2}{3} + \gamma^2\sqrt{W} - \frac{F(1 - 4\alpha^2\gamma^2 F)}{2\sqrt{W}}. \tag{26}$$

As we can see in Fig. 1, this system admits only two critical points (F_0, θ_0), that is,

(i) Point A: $(0, -\sqrt{3\gamma^2})$

(ii) Point B: $(0, +\sqrt{3\gamma^2})$

The value $F = 0$ is attained when the scale factor go to infinity, once $F = 2B_0^2/a^2$ that is for $\varrho = \gamma^2$ which corresponds to the infinite past and to the infinite future [see Eq. (23)]. Looking at the plot in the phase space one finds the expected result that at $t \to -\infty$ the system is unstable and at $t \to +\infty$ it is stable. Thus, the set of all integral trajectories of the solutions of our dynamical system has the same behavior: it starts at the infinite past with a very large volume collapses pass a minimum where θ vanishes and then start to increase.

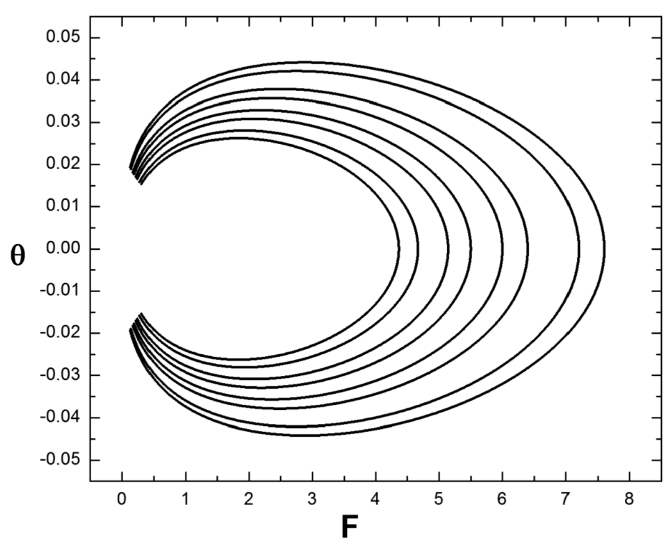

FIG. 1. Space of phase $(\theta \times F)$.

A. Domain of F

In the case of a magnetic configuration of the Born-Infeld theory the quantity $U = 1 + F/2\gamma^2$ may attain any value. This means that the field $F = 2B^2$ can take arbitrary large values. On the other hand, in the extended theory presented here the corresponding quantity $W = 1 + F/2\gamma^2 - \alpha^2F^2$ restricts the domain of the magnetic field in the range (Fig. 2)

$$0 < F < \frac{1 + \sqrt{1 + 16\alpha^2\gamma^4}}{4\alpha^2\gamma^2}.$$

As a consequence of this the density of energy is also restrained. Indeed the energy can assume values only in the domain

$$1 < \frac{\varrho}{\gamma^2} < \sqrt{1 + \frac{1}{16\alpha^2\gamma^4}}.$$

Let us note that a very small value of α is crucial in order that the density of energy may attain values that are far from the minimum γ^2 (Fig. 4).

B. Special values of the field

There are some special values of F that identify some important properties concerning not only the magnetic field itself but the values of the scale factor. Let us describe some of them here concerning three important values F that we will name F_c, F_n, and F_b. They are defined, respectively, by the properties:

(i) F_c is the value in which the density of energy attains an extremum and consequently—using Eq. (15)—at

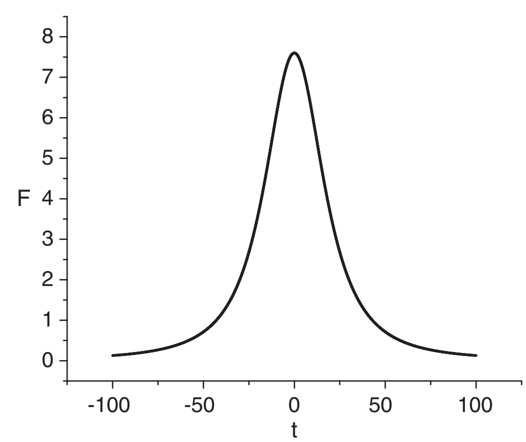

FIG. 2. Time evolution of the field F. It is inversely proportional to the scale factor. Note that it has a maximum precisely at the bounce.

this point the energy density and the pressure takes the values such that $\varrho + p = 0$;

(ii) F_n is the value in which the quantity W pass the value 1 for the first time (it will end at this same value 1 in the asymptotic regime $F = 0$);

(iii) F_b is the value at the bouncing, where θ vanishes. See Eq. (15).

The values of these quantities are given by

$$F_c = \frac{1}{4\alpha^2\gamma^2}, \tag{27}$$

$$F_n = \frac{1}{2\alpha^2\gamma^2}, \tag{28}$$

and F_b satisfies the equation of general relativity

$$\varrho - \frac{3}{a^2} = \frac{\theta^2}{3},$$

that is, in our case, setting $Z = F_b$

$$\alpha^2\gamma^4 Z^2 + qZ - \gamma^4 = 0, \tag{29}$$

where we have defined

$$q \equiv \frac{9 - B_0^2\gamma^2}{2B_0^2}.$$

From these expressions it follows that the order of these quantities is given by $F_c < F_n < F_b$ for arbitrary values of the constant B_0.

Note that once the scale factor has a minimum at F_b the domain of the values of the field that belong to the region

$$F_b < F < X_+$$

is forbidden.

C. Behavior of the scale factor: Bouncing configuration

When F is bounded from zero to its maximum value F_b the scale factor varies in the domain

$$a_b < a(t) < \infty,$$

where

$$a_b^4 = \frac{2B_0^2}{Z}$$

with

$$Z = \frac{1}{4\alpha^2\gamma^2}(M + \sqrt{M^2 + 16\alpha^2\gamma^4})$$

and

$$M \equiv 1 - \frac{9}{B_0^2\gamma^2}.$$

We recognize from this expression that it is the presence of α the responsible for the existence of a minimum of the scale factor. In the case of Born-Infeld theory, that is, for $\alpha = 0$, the scale factor attains the value zero, implying the existence of a singularity of the cosmological metric.

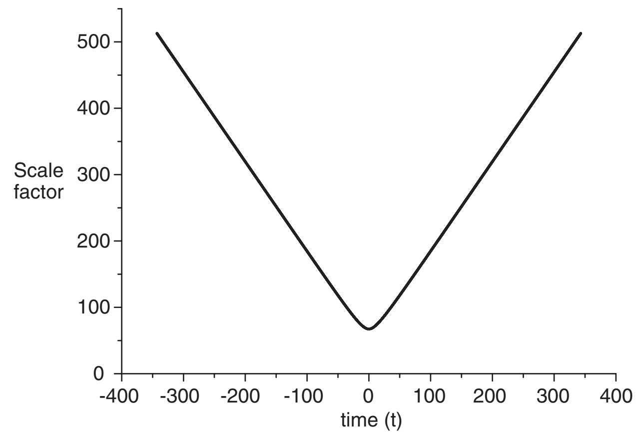

FIG. 3. Graphic of scale factor with time, obtained by a series of iterations starting from the minimum a_b.

The requirement for the existence of a minimum of $a(t)$ is equivalent to the inequality $F_b > F_c$ which imposes a condition on the minimum possible value of the intensity of the field given by

$$B_0^2 > B_{\rm cr}^2,$$

where

$$B_{\rm cr}^2 = \frac{18}{\gamma^2(1 + 16\alpha^2\gamma^4)}.$$

In other words, if the value of the field is lower than such critical value $B_{\rm cr}$ there is no minimum: a singularity becomes inevitable [Fig. 3].

D. Behavior of the energy and the pressure

The expression of the energy density is given by

$$\rho = \gamma^2\left(1 + \frac{B_0^2}{\gamma^2 a^4} - \frac{4\alpha^2 B_0^4}{a^8}\right)^{1/2}. \tag{30}$$

Using the conservation equation (15) it follows that there exists three extremum for the density: two maxima at the value $F = F_c$ and at the bouncing. We have already noted that the behavior is apparently counterintuitive, once at the bouncing the energy density has a minimum and the others two points are maximum. At the past infinity and at the future infinity when the scale factor increases without limit, the density goes to a constant, that is,

$$0 \leq \varrho < \gamma^2.$$

We note that the pressure is always negative (Fig. 5). The quantity that controls the time evolution of the density, in the expanding Universe, is $\varrho + p$. This quantity changes the sign during the evolution at the points $t = t_c$.

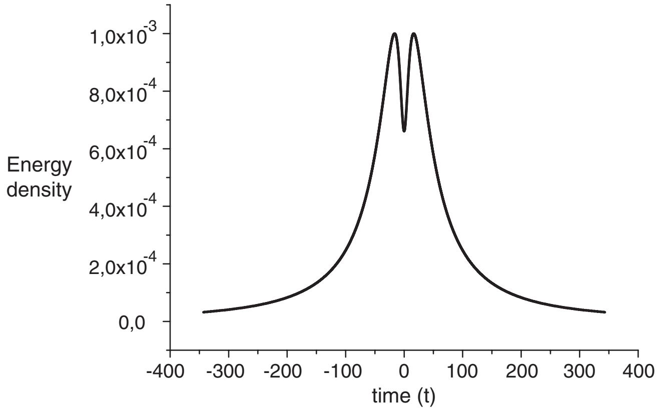

FIG. 4. Time evolution of the energy density. Asymptotically when $t \rightarrow \pm\infty$, the energy density converges to the constant γ^2.

VII. THE INEVITABILITY OF THE PRESENCE OF THE COSMOLOGICAL CONSTANT IN THE GENERALIZED BORN-INFELD ELECTRODYNAMICS

In their paper Born and Infeld consider the Lagrangian as given by

$$L = -\gamma^2\sqrt{U} + \gamma^2, \tag{31}$$

where $U = 1 + F/2\gamma^2$. The extra term γ^2 was not important for the electrodynamics and it was added only to set the energy content of the field to go asymptotically to zero. In other words, to eliminate the possibility that the energy-momentum tensor of the electromagnetic field reduces to a cosmological constant. One should wonder if a similar procedure could be made in the context of our generalization when the factor U is changed for W given by

$$W = 1 + F/2\gamma^2 - \alpha^2F^2.$$

The answer is a definite no. Let us explain this.

Let us start by supposing that to our Lagrangian a similar extra term is added. This means that the density of energy in this hypothetical case assumes the expression

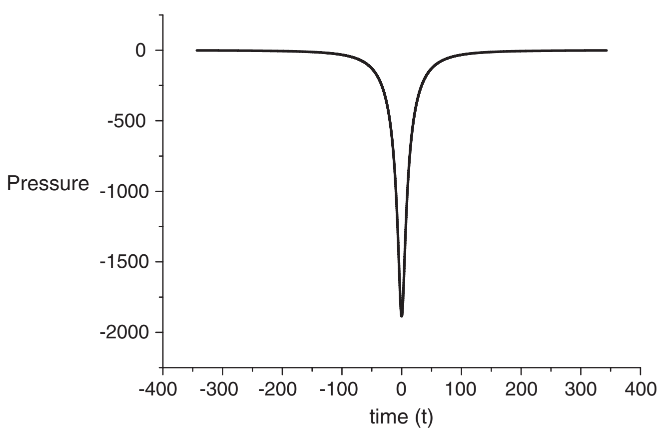

FIG. 5. Time evolution of pressure. As the energy density, pressure asymptotically goes to a constant: $p \rightarrow -\rho$.

$$\tilde{\varrho} = \gamma^2\sqrt{W} - \gamma^2.$$

The question then appears: is such expression positive definite? In the case of Born-Infeld the answer is yes, but this is not the case in our generalized electrodynamics.

To show this let us consider three important values of the field F that we will name F_c, F_n, and F_b.

From the expression of W it follows that if one insists in eliminate the cosmological constant as it is done in the standard Born-Infeld case, the price to pay is too high: nonpositivity of the density of energy in the domain limited by (F_n, F_b). It then follows that in the extended Born-Infeld electrodynamics presented here, the asymptotic vacuum regime in a magnetic universe is inevitably [14].

VIII. CONCLUSION

The main motivation of the present work concerns the necessity to construct a theory in which the combined long-range fields shows a regular behavior. We start from the generic idea that guided Max Born to the attempt to construct a new nonlinear electrodynamics in order to display a regular behavior of the electric field of a charged body. The application of the Born-Infeld nonlinear theory in the cosmological context within the framework of general relativity shows singular behavior that is not acceptable in the light of their original proposal. We are thus conducted to an alternative form for the dynamics which retains all nice properties on the pure electromagnetic sector of the Born-Infeld theory without its main difficulty when dealing within cosmology.

The model presented here displays many regular properties that should be worth further investigation. In particular, it has two basic properties that a classical theory should pursue:

(i) Regular behavior of the electric field of a charged body;
(ii) Regular behavior of the combined fields of electrodynamics and gravity in a cosmological framework.

We have shown that in the magnetic universe representing a spatially homogeneous and isotropic Friedmann-Lemaître geometry, there is no room for a singular behavior—contrary to the Einstein-Maxwell and Einstein-Born-Infeld description of electrodynamics and gravity. The main properties of this cosmological scenario are the following:

(i) The geometry of the Universe is driven by a magnetic fluid;
(ii) At the primordial phase this fluid mimics a cosmological constant in the unstable state of a very large scale factor;
(iii) As a consequence, the universe starts to collapse;
(iv) The collapse stops when the universe attains the minimum value of the scale factor $a_{\min} = a_b$;

(v) The final configuration is identical to a perfect fluid with the equation of state of the vacuum $\varrho + p = 0$, and the volume goes to infinity.

Thus, we can represent this model as a vacuum-to-vacuum configuration. In order to make such model more realistic, one must introduce ordinary matter that contributes to the total density of energy with a term which, for instance, depends on the scale factor as a^{-3}. This term has to be added to our scenario presented in this paper. It is not difficult to see that a phase controlled by such dust matter interpolates between phase controlled by a^{-4} and the accelerating cosmological vacuum. We shall address this elsewhere.

ACKNOWLEDGMENTS

J. S. is supported by CNPq. M. N acknowledges the support of FAPERJ and CNPq. A. N. A. is supported by CnPq. We acknowledge FINEP for financial support. We thank our colleagues from ICRA for their comments on this paper and P. Diniz Batista for his support on the development of the algorithm for the graphics.

APPENDIX A: STATIC AND SPHERICALLY SYMMETRIC ELECTROMAGNETIC SOLUTION AND THE ASYMPTOTIC REGIME

In the present work we made an analysis of the modification of electrodynamics in a cosmological context. We are not arguing that these effects are due to the response of the Universe to local electrodynamics. Some decades ago Wheeler and Feynman made a conjecture that local properties of electrodynamics (e.g., the Lienard-Wiechert potential) may just be a consequence of such cosmic response inducing the elimination of advanced fields. However, if one takes these modifications as a global change of electrodynamics, we should check consistency of the theory with laboratory properties. Let us look into the case of the static electric field generated by a point charged particle in the new theory. For a general nonlinear Lagrangian $L = L(F)$, the equation of motion for the point charge reduces to

$$r^2 L_F E(r) = \text{const.}$$

In the case of the Lagrangian of our theory we get

$$E^2\left(\frac{1 + 8\alpha^2\gamma^2E^2}{-\gamma^2 + E^2 + 4\alpha^2\gamma^2E^4}\right)^{1/2} = \frac{q}{r^2}. \tag{A1}$$

1. Asymptotic regime

Let us make an extra comment on the above case of a point charge particle at spatial infinity. The energy-momentum tensor has the form:

$$T_{\mu\nu} = -Lg_{\mu\nu} - 4L_F F_{\mu\alpha}F^{\alpha}\nu, \tag{A2}$$

which in the present case $F_{01} = E(r)$ is

$$T^0_0 = T^1_1 = -L - 4E^2L_F \qquad T^2_2 = T^3_3 = L. \tag{A3}$$

In the asymptotic regime the energy-momentum tensor takes the value

$$T^0_0 = T^1_1 = T^2_2 = T^3_3 = \gamma^2, \tag{A4}$$

which mimics a Λ term.

If we add an extra term in the Lagrangian we could eliminate the residual constant field at infinity. In the case of Maxwell electrodynamics such ambiguity of choice does not arise due to its linearity. However, for a nonlinear electromagnetic theory a new possibility occurs which concerns the geometrical structure at infinity. This means that for the nonlinear electrodynamics the fact that at infinity the field is a constant does not imply that it vanishes. Such a property can be translated in a formal question, that is, what is the asymptotic regime of the geometry of space-time: Minkowski or de Sitter?

In classical linear electrodynamics the answer to that question was known and did not pose any ambiguity. This is no longer so if a nonlinear electromagnetic field is combined with the equations of general relativity. The possibility of the de Sitter structure must be considered. In theories in which a solution distinct from zero for the equation $L_F = 0$ exists, such a question has to be investigated combined with cosmology.

APPENDIX B: THE FUNDAMENTAL STATE

A simple look into the equation of motion shows the existence of a particular solution such that its energy distribution is the same as the one in the vacuum fundamental state represented by an effective cosmological constant. Indeed, the equation of motion is given by

$$(L_F F^{\mu\nu})_{;\nu} = 0. \tag{A5}$$

Consider the particular solution $F = F_0 =$ constant such that

$$4\alpha^2\gamma^2F_0 - 1 = 0.$$

This is the condition that satisfies the equation of motion since L_F vanishes at this value F_0. In this state the corresponding energy-momentum takes the form

$$T_{\mu\nu} = \Lambda g_{\mu\nu},$$

where

$$\Lambda = -L(F_0) = \gamma^2\left(1 + \frac{1}{16\alpha^2\gamma^4}\right)^{1/2}.$$

This property is typical of the nonlinear electrodynamics, since the linear Maxwell theory is not able to display such particular solution.

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