World Scientific
www.worldscientific.com

Chiral symmetry breaking as a geometrical process

E. Bittencourt,[*,†,‡,§] S. Faci[*,¶] and M. Novello[*,‖]

[*]*Instituto de Cosmologia, Relatividade e Astrofísica ICRA-CBPF,*
Rua Doutor Xavier Sigaud, 150, Urca, CEP 22290-180, Rio de Janeiro, RJ, Brazil
[†]*Sapienza Università di Roma, Dipartimento de Fisica,*
P.le Aldo Moro 5-00185, Rome, Italy
[‡]*ICRANet, Piazza della Repubblica, 10-65122, Pescara, Italy*
[§]*eduardo.bittencourt@icranet.org*
[¶]*sofiane@cbpf.br*
[‖]*novello@cbpf.br*

This paper is an extension for spinor fields the recently developed *Dynamical Bridge* formalism which relates a linear dynamics in a curved space to a nonlinear dynamics in Minkowski space. This leads to a new geometrical mechanism to generate a chiral symmetry breaking without mass, providing an alternative explanation for the absence of right-handed neutrinos. We analyze a spinor field obeying the Dirac equation in a curved space which is constructed by its own current. This way, both chiralities of the spinor field satisfy the same dynamics in the curved space. Afterward, the dynamical equation is re-expressed in terms of the flat Minkowski space and then each chiral component behaves differently. The left-handed part of the spinor field satisfies the Dirac equation while the right-handed part is trapped by a Nambu–Jona-Lasinio type potential.

Keywords: Differential geometry; nonlinear spinors; neutrino physics.

PACS numbers: 02.40.Ky, 14.60.St, 11.30.Rd

1. Introduction

This work is related to the so-called *Dynamical Bridge* (DB) formalism. The latter states that fundamental fields can satisfy equivalent dynamical equations in different background geometries, which was already applied to scalar and vector fields.[1–3] Namely, a given nonlinearity of the field in the flat space can be "hidden" in the curvature of a curved space. It is worth noting that despite the DB method deals with curved spaces, these have no gravitational character and are completely determined by the dynamical field. For instance, in Ref. 4, it is possible to see how gravity can be dynamically described by scalar fields, manifesting itself through a

E. Bittencourt, S. Faci & M. Novello

curved space which does not have a proper dynamics, contrary to what happens in Einstein's theory. Another example concerns electromagnetic fields (cf. Ref. 5), where part of the electromagnetic interaction of the leptons is geometrized. In this vein, the DB offers a uniform geometrical description in the sense that it induces a curved geometry for all kinds of nonlinear interactions.

Here we shall demonstrate that such procedure also holds for spinor fields. In particular, we show that the self-interacting term of the Nambu–Jona-Lasinio (NJL) equation — which appears commonly as a phenomenological regime in quantum chromodynamics (QCD) based on chiral symmetry breaking (see historical details in Ref. 6) — can be derived from the minimal coupling of a Dirac field with a curved geometry. Therefore, this DB goes together with the chiral symmetry breaking even for massless spinor fields if the coupling constant is sufficiently large. We thus have a new and geometrical explanation for the nonobservation of the right-handed neutrinos.

In our proposal, we consider a massless spinor field Ψ satisfying the Dirac equation $i\hat{\gamma}^\mu \hat{\nabla}_\mu \Psi = 0$ in a curved background without any gravitational character. This curved geometry represents the modification of the flat space caused only by the presence of the field Ψ. The latter modifies the Minkowski metric through the simplest possible way by means of its vector J^μ and axial I^μ currents. Then, we write the curved "spinorial" metric, by introducing a vector H^μ which is a linear combination of J^μ and I^μ, as follows

$$\hat{g}^{\mu\nu} = \eta^{\mu\nu} + 2\alpha H^\mu H^\nu \,, \tag{1}$$

where the arbitrary function α depends on the scalars, $A \equiv \bar{\Psi}\Psi$ and $B \equiv i\bar{\Psi}\gamma_5\Psi$. Afterward, we rewrite this dynamics in Minkowski space (equipped with the metric $\eta_{\mu\nu}$). In an appropriate regime of α, this leads to the two following equations

$$i\gamma^\mu \partial_\mu \Psi_L = 0 \,, \tag{2}$$

$$i\gamma^\mu \partial_\mu \Psi_R + s(A + iB\gamma_5)\Psi_L = 0 \,, \tag{3}$$

where s depends on α and its derivatives, $\Psi_L \equiv (1/2)(\mathbf{1} - \gamma_5)\Psi$ and $\Psi_R \equiv (1/2)(\mathbf{1} + \gamma_5)\Psi$ are respectively, the left-handed and right-handed chirality components of Ψ, with $\Psi = \Psi_L + \Psi_R$. The left-handed component still obey the Dirac dynamics in Minkowski space while the right-handed component verifies Eq. (3), which is a generalization of the NJL dynamics. The curvature of the effective space goes to the nonlinearity of NJL dynamics in Minkowski space, but only for the right-handed component of the spinor field. This introduces a new answer to the experimentally observed chiral symmetry breaking of neutrinos and stands as an alternative for the Standard Model (SM) interpretation which assumes that right-handed neutrinos do not exist, i.e. they do not interact through the SM forces. Further extensions assume the right-handed neutrinos have very large masses which makes them invisible.

This paper is organized as follows. The next section introduces the mathematical background to understand the DB formalism and particularly how the curved

space–time is built with spinors. Section 3 expands the DB for spinor fields and exhibits how Dirac's dynamics in a curved space leads to a nonlinear dynamics in Minkowski space. In Sec. 4, an important regime of the resulting nonlinear dynamics is explored yielding a generalized NJL equation. Finally, Sec. 5 summarizes our concluding remarks.

2. Mathematical Compendium for the Dynamical Bridge

In order to properly construct the DB for spinor fields, let us describe some mathematical ingredients. In this section, we focus on the algebraic relations that the Dirac matrices have to satisfy in both spaces.

According to the field theory formulation of a spin-2 field,[7] any metric tensor $\hat{g}^{\mu\nu}$ can always be decomposed into

$$\hat{g}^{\mu\nu} = g^{\mu\nu} + \Sigma^{\mu\nu}, \tag{4}$$

where $g^{\mu\nu}$ is some background metric and $\Sigma^{\mu\nu}$ is a rank-2 tensor field. Under this form the inverse metric $\hat{g}_{\mu\nu}$ should be expressed by an infinite series. However, the metric $\hat{g}^{\mu\nu}$ admits an inverse with the same binomial form if we impose the condition $\Sigma^{\mu\nu}\Sigma_{\nu\lambda} = p\delta^{\mu}_{\lambda} + q\Sigma^{\mu}{}_{\lambda}$, where p and q are arbitrary functions of the coordinates. Throughout the text, we set the background metric $g_{\mu\nu}$ to be the Minkowski one $\eta_{\mu\nu}$ in arbitrary coordinate systems. Though this might appear restrictive, it can be generalized to arbitrary curved backgrounds.[8]

With a generic spinor field Ψ (more rigorously, a complex bispinor spin-1/2 representation of the Lorentz group), we construct two Hermitian scalars $A \equiv \bar{\Psi}\Psi$ and $B \equiv i\bar{\Psi}\gamma_5\Psi$, where the adjoint field is given by $\bar{\Psi} = \Psi^{\dagger}\gamma^0$. We also define the associated vector and axial currents, respectively, as $J^{\mu} \equiv \bar{\Psi}\gamma^{\mu}\Psi$ and $I^{\mu} \equiv \bar{\Psi}\gamma^{\mu}\gamma_5\Psi$. The γ^{μ}'s are the Dirac matrices which satisfy the closure relation of the Clifford algebra

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}\mathbf{1}, \tag{5}$$

where $\mathbf{1}$ is the unity matrix of the algebra. Using the Pauli–Kofink identity, see the appendix of Ref. 9, which states that for any element Q of the Clifford algebra one has

$$(\bar{\Psi}Q\gamma_{\lambda}\Psi)\gamma^{\lambda}\Psi = (\bar{\Psi}Q\Psi)\Psi - (\bar{\Psi}Q\gamma_5\Psi)\gamma_5\Psi, \tag{6}$$

the following relations are easily derived

$$J^2 = -I^2 = A^2 + B^2, \quad J_{\mu}I^{\mu} = 0, \tag{7}$$

where we have denoted $X^2 \equiv \eta_{\mu\nu}X^{\mu}X^{\nu}$ for the vectorial objects.

The DB method consists in writing $\Sigma^{\mu\nu}$ of the curved space–time metric (4) in terms of the dynamical field which is Ψ. Assuming a second-order relation of $\Sigma^{\mu\nu}$ in term of the currents, its most general form, thanks to the relations (7), reads

$$\Sigma^{\mu\nu} \doteq 2\alpha H^{\mu}H^{\nu}, \tag{8}$$

where $H^\mu \doteq J^\mu + \epsilon I^\mu$ is a linear combination of the currents, ϵ is an arbitrary constant and α is an arbitrary function of the scalars A and B. The factor 2 is conventional. Equations (6) and (7) leads to the following identities

$$H_\mu \gamma^\mu \Psi = (\mathbf{1} + \epsilon \gamma_5)(A + iB\gamma_5)\Psi \quad \text{and} \quad H^2 = (1 - \epsilon^2)J^2 \,. \tag{9}$$

Finally, using the relation $\hat{g}^{\mu\nu}\hat{g}_{\nu\lambda} = \delta^\mu_{\ \lambda}$, the metric tensor and its inverse can be written as

$$\begin{aligned}
\hat{g}^{\mu\nu} &= \eta^{\mu\nu} + 2\alpha H^\mu H^\nu \,, \\
\hat{g}_{\mu\nu} &= \eta_{\mu\nu} - \frac{2\alpha}{1 + 2\alpha H^2} H_\mu H_\nu \,.
\end{aligned} \tag{10}$$

This kind of binomial metrics (10) can be seen as an example of the disformal transformations introduced by Bekenstein[10,11] and, since then, they have been investigated in many contexts like bimetric theories,[12] cosmic inflation,[13] analogue models[14,15] and others.

Note that we are dealing with curved spaces without any gravitational character. Thus, the metric (10) might appear artificial, however, this is not important once the DB uses the curved metric merely as a "tool" to incorporate nonlinear interactions in the curvature of the space. Furthermore, the physical aspects of this mathematical map become clear when a physical phenomenon can be explained through geometrical objects. We will see hereafter that, the DB yields a physical phenomenon like the chiral symmetry breaking for massless neutrinos.

From now on, we make use of the tetrad formalism[a] (or vierbeins), in order to perform the DB by changing the tangent (spinor) space and the physical space–time simultaneously. This allows to simplify the calculations since we are dealing with spinors in curved background. The Lagrangian and the dynamical equations for Ψ must then be invariant not only under diffeomorphisms, but also under local Lorentz transformations acting on the tangent space.

Let us now rewrite the objects defined previously in terms of the tetrads. We define two tetrad bases $e^\mu_{\ A}$ and $\hat{e}^\mu_{\ A}$ which relate the tangent space provided with the metric η_{AB} to the physical spaces endowed with the two metrics $\eta_{\mu\nu}$ and $\hat{g}_{\mu\nu}$. The two bases satisfy the following relations

$$\begin{aligned}
\hat{g}^{\mu\nu} &= \eta^{AB} \hat{e}^\mu_{\ A} \hat{e}^\nu_{\ B} \,, \\
\eta^{\mu\nu} &= \eta^{AB} e^\mu_{\ A} e^\nu_{\ B} \,.
\end{aligned} \tag{11}$$

The Greek indices refer to the physical spaces and are lowered and raised by the corresponding metric ($\eta_{\mu\nu}$ or $\hat{g}_{\mu\nu}$). The capital Latin indices refer to the tangent space and are lowered and raised by $\eta_{AB} = \text{diag}(1, -1, -1, -1)$. The inverse tetrad bases $e_\mu^{\ A}$ and $\hat{e}_\mu^{\ A}$ should satisfy $e_\mu^{\ A} e^\nu_{\ A} = \hat{e}_\mu^{\ A}\,\hat{e}^\nu_{\ A} = \delta^\nu_\mu$ and $e_\mu^{\ A} e^\mu_{\ B} = \hat{e}_\mu^{\ A}\,\hat{e}^\mu_{\ B} = \delta^A_B$. Furthermore, any vector X^μ (or $\hat{X}^\mu$) in the space–time $\eta_{\mu\nu}$ (or $\hat{g}_{\mu\nu}$) has a

---

[a]This formalism was introduced by Cartan in Ref. 16 and generalized by Weyl in Ref. 17 to adequately treat spinors in curved spaces.

counterpart $X^A \equiv e^A_\mu X^\mu$ (or $\hat{X}^A \equiv \hat{e}^A_\mu \hat{X}^\mu$) in the tangent space. In particular, for the Dirac matrices we assume that

$$\gamma^A = \hat{e}_\mu{}^A \hat{\gamma}^\mu = e_\mu{}^A \gamma^\mu \,, \tag{12}$$

where γ^A's are the constant Dirac matrices. Note that the Dirac matrices $\hat{\gamma}^\mu$ also verify the closure relation of the Clifford algebra corresponding to the metric $\hat{g}^{\mu\nu}$, that is

$$\{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} = 2\hat{g}^{\mu\nu}\mathbf{1} \,. \tag{13}$$

For consistency with Eq. (5) and Eqs. (10)–(13), the two tetrad bases must be related. However, this relation is not unique. So, inspired in the form of the metric (10), we are lead to

$$\hat{e}^\mu{}_A = e^\mu{}_A + \beta H_A H^\mu \,. \tag{14}$$

Using Eqs. (10) and (11), we have the constraint $\alpha = 2\beta/(2 + \beta H^2)$. Therefore, the inverse tetrad bases are related according with

$$\hat{e}_\mu{}^A = e_\mu{}^A - \frac{\beta}{1 + \beta H^2} H^A H_\mu \,. \tag{15}$$

That is all we need about spinor fields in curved spaces and tetrad formalism in order to construct the bridge between the dynamics. In the next section we shall focus on the map itself.

3. Dynamical Bridge for Spinor Fields: From Dirac to a Nonlinear Dynamics

Now we describe how the DB works for spinor fields. As mentioned before, we start with the linear Dirac equation in an effective curved background, then we expand the formulas to reach a new and different, though mathematically equivalent, dynamics in the Minkowski background. The resulting equation is a generalization of the NJL dynamics.

In the effective curved geometry given by (10), the Dirac equation for the spinor field[b] reads

$$i\hat{\gamma}^\mu \hat{\nabla}_\mu \Psi = 0 \,, \tag{16}$$

where $\hat{\nabla} \equiv \partial_\mu - \hat{\Gamma}_\mu$ and the Fock–Ivanenko connection $\hat{\Gamma}_\mu$ is given by

$$\hat{\Gamma}_\mu = -\frac{1}{8}([\hat{\gamma}^\alpha, \hat{\gamma}_{\alpha,\mu}] - \hat{\Gamma}^\rho_{\alpha\mu}[\hat{\gamma}^\alpha, \hat{\gamma}_\rho]) \,. \tag{17}$$

Note that a comma means partial derivative, $\hat{\Gamma}^\rho_{\alpha\mu}$ is the Christoffel symbol constructed with $\hat{g}_{\mu\nu}$ and the squared brackets represent the commutator operator. Introducing the tetrads allows to rewrite the Fock–Ivanenko connection (17) as

$$\hat{\Gamma}_\mu = \hat{e}_\mu{}^A \hat{\Gamma}_A \quad \text{with} \quad \hat{\Gamma}_A = -\frac{1}{8} \hat{\gamma}_{BCA}[\gamma^B, \gamma^C] \,, \tag{18}$$

[b]Unlike what was done in Ref. 18, here Ψ remains the same in both spaces.

where $\hat{\gamma}_{ABC}$ is called spin connection and

$$\hat{\gamma}_{ABC} = \frac{1}{2}(\hat{C}_{ABC} - \hat{C}_{BCA} - \hat{C}_{CAB}) \quad \text{and} \quad \hat{C}_{ABC} = -\hat{e}_{\nu A}\hat{e}^\mu{}_{[B}\hat{e}^\nu{}_{C],\mu}. \qquad (19)$$

It is easy to see that $\hat{C}_{ABC} = -\hat{C}_{ACB}$ and, consequently, $\hat{\gamma}_{ABC} = -\hat{\gamma}_{BAC}$. To go further we use the relation (14) to expand (19), as follows

$$\hat{C}_{ABC} = -\frac{1}{1+\beta H^2}(H_A\beta_{,[B}H_{C]} + \beta^2 H_A H_{[B}\dot{H}_{C]} - \beta H_A H_{[C,B]}) + \beta H_{A,[B}H_{C]}$$

$$-\frac{\beta^2}{2(1+\beta H^2)}H_A H^2_{,[B}H_{C]}, \qquad (20)$$

where $\dot{H}_A$ holds for $H_{A,B}H^B$. Thus, we compute the spin connection

$$\hat{\gamma}_{ABC} = \hat{C}_{ABC} + \frac{\beta}{2}\left[\frac{\beta H^2}{1+\beta H^2}(H_A H_{[B,C]} + H_B H_{[C,A]} - H_C H_{[B,A]})\right.$$

$$\left. + H_{A,[C}H_{B]} + H_{[B,C]}H_A\right]. \qquad (21)$$

The Dirac equation (16) reads

$$i\hat{\gamma}^A(\hat{\partial}_A - \hat{\Gamma}_A)\Psi = i\gamma^A\left(\partial_A + \beta H_A H^B\partial_B + \frac{1}{8}\hat{\gamma}_{BCA}[\gamma^B, \gamma^C]\right)\Psi = 0, \qquad (22)$$

where $\hat{\partial}_A \equiv \hat{e}^\mu{}_A\partial_\mu = \partial_A + \beta H_A H^B\partial_B$. To proceed we need to compute the expression $\hat{\gamma}_{BCA}\gamma^A\gamma^B\gamma^C$. This turns out to contain only two nonvanishing terms,[c] which are

$$\hat{\gamma}_{BCA}\epsilon^{ABC}{}_D = \frac{\beta^2 H^2}{2(1+\beta H^2)}H_A H_{[B,C]}\epsilon^{ABC}{}_D \qquad (23)$$

and

$$\hat{\gamma}_{BCA}\eta^{AB}\gamma^C = \frac{1}{1+\beta H^2}\left[\dot{\beta}H_A\gamma^A - H^2(\beta\cdot\gamma) + \beta^2 H^2\dot{H}_A\gamma^A - \beta^2\dot{H}_A H^A H_B\gamma^B\right.$$

$$\left. -\frac{\beta}{4}(H^2\cdot\gamma) + \frac{\beta}{2}\dot{H}_A\gamma^A\right] - \frac{\beta}{4}(H^2\cdot\gamma) - \frac{\beta^2}{2(1+\beta H^2)}\dot{H}^2 H_A\gamma^A$$

$$+ \frac{\beta^2}{2(1+\beta H^2)}H^2(H^2\cdot\gamma). \qquad (24)$$

We have used the shortcuts $\dot{\beta} \equiv \beta_{,A}H^A$, $\beta\cdot\gamma \equiv \beta_{,A}\gamma^A$ and $H^2\cdot\gamma \equiv 2H^A H_{A,B}\gamma^B$. Equation (22) does not match any well-known dynamics for a nonlinear spinor field in flat space. To get a simpler equation, we should assume a specific value for ϵ. From Eq. (9), setting $\epsilon^2 = 1$, it implies that $H^2 = 0$ drastically simplify the

---

[c]To do this, we use the following algebraic identity for the Dirac matrices $\gamma^A\gamma^B\gamma^C = \eta^{AB}\gamma^C + \eta^{BC}\gamma^A - \eta^{AC}\gamma^B + i\epsilon^{ABC}{}_D\gamma^D\gamma_5$.

dynamical equations.[d] Let us assume $\epsilon = -1$ for later convenience. Consequently, the expression (23) vanishes and (24) reduces to

$$\hat{\gamma}_{BCA}\eta^{AB}\gamma^C = \dot{\beta}H \cdot \gamma + \frac{\beta}{2}\dot{H} \cdot \gamma \,. \tag{25}$$

Therefore, the spinor field equation (16) written in the curved space without any gravitational character, becomes the following equation in the flat space

$$i\left(\gamma^\mu\partial_\mu + \beta\gamma^\mu H_\mu H^\nu\partial_\nu + \frac{\beta}{4}\dot{H}_\mu\gamma^\mu + \frac{\dot{\beta}}{2}H_\mu\gamma^\mu\right)\Psi = 0 \,. \tag{26}$$

This is a nonlinear dynamical equation for the spinor field in the Minkowski space. The terms originated from the effective curved space connection are interpreted as self-interacting terms. As it is, this equation does not fit any known nonlinear dynamics for spinor fields,[19] because the propagating terms proportional to β have nonphysical interpretation. Nonetheless, more simplifications can be applied and some remarkable results emerge when we examine closely the coupling parameter β and look for particular solutions.

4. Breaking Chiral Symmetry Without Mass

In the SM of particles, the neutrino is represented by a massless spinor field. It means that the results of the DB developed above are particularly applicable for the scenarios where this particle is involved. In this vein, let us remark that in the Lagrangian of the SM, the right-handed neutrinos are not present since weak interactions couple only to the left-handed neutrinos and due to the symmetries of the model a mass term for neutrinos is not allowed. Yet the 1998's experiment, Super-Kamiokande, and various other independent and different experiments have detected the family oscillation of this particle.[20] However, from the minimal coupling principle applied to Minkowski space, this can be explained if the neutrinos are lightly massive, breaking the chiral symmetry. But a fundamental question raises: where are the not yet observed right-handed neutrinos?

Nowadays, there are two possible ways to answer this question: either they just do not exist and then massive neutrinos are of Majorana type, which is not really satisfactory; or, they exist but do not interact weakly. We should also remark that due to the mixing of chiralities caused by the mass term, it was led to assume that right-handed neutrinos are heavy enough in order to hide them from observations. Then, the chiral symmetry is broken "by hand" and it does not really explain why such neutrinos are so massive. Several experiments have unsuccessfully tried direct or indirect detection of such neutrinos, as one can see in Refs. 21 and 22 and references therein.

Notwithstanding, this work provides a third possible explanation for the chiral symmetry breaking according to a geometrical viewpoint. In the regime $\dot{\beta} \gg \beta$,

---

[d]It should be remarked that this does not imply $J^2 = 0$ which otherwise would be too restrictive.

when the nonlinear propagating terms of Eq. (26) can be indeed neglected, it reduces to

$$i\left(\gamma^\mu\partial_\mu + \frac{\dot\beta}{2}H_\mu\gamma^\mu\right)\Psi = 0\,. \tag{27}$$

Substituting the expression for $H_\mu\gamma^\mu\Psi$ from Eq. (9), we obtain

$$i\gamma^\mu\partial_\mu\Psi + i\frac{\dot\beta}{2}(\mathbf{1}-\gamma_5)(A+iB\gamma_5)\Psi = 0\,. \tag{28}$$

This equation is very similar to the NJL equation for a spinor field propagating in Minkowski space–time. This means that the nonlinear self-interacting term can be interpreted as a modification of the space–time structure. Up to this point, our method seems to present two equivalent dynamical equations though written in two different spaces. But there is a "hidden" property that appears in the flat case when we decompose the spinor Ψ into its two opposite chiralities. That is

$$\Psi = \Psi_L + \Psi_R = \frac{1}{2}(\mathbf{1}-\gamma_5)\Psi + \frac{1}{2}(\mathbf{1}+\gamma_5)\Psi\,,$$

where Ψ_L and Ψ_R are components of the spinor field representing the left- and the right-handed chiralities. Therefore, Eq. (28) splits into two distinct parts

$$i\gamma^\mu\partial_\mu\Psi_L = 0\,, \tag{29}$$

$$i\gamma^\mu\partial_\mu\Psi_R + i\dot\beta(A+iB\gamma_5)\Psi_L = 0\,. \tag{30}$$

From these equations, it follows the remarkable result: each chiral component Ψ_L and Ψ_R satisfies a different dynamical equation in the Minkowski space. The left-handed component propagates as a free Dirac field when the right-handed component is trapped by the self-interacting term. If the coupling parameter $\dot\beta$ is sufficiently large, the right-handed component needs very high energies to be detected. This leads to a new and geometrical explanation for the nonobservation of the right-handed neutrinos.

5. Concluding Remarks

This paper completes a sequence of works dealing with the DB, a mathematical equivalence between different dynamical equations of a given field (scalar, vector and now spinor) defined in different space–times. In particular, we have shown that a massless spinor field satisfying the linear Dirac equation defined in a curved geometry is equivalent to the same spinor field in the Minkowski space with its right-handed component satisfying a modified NJL equation and the left-handed component still satisfying the massless Dirac equation. If the coupling parameter is sufficiently large, the right-handed component needs very high energies to be detected. This leads to a new and geometrical explanation for the nonobservation of the right-handed neutrinos.

It is important to notice that the set of solutions of Eq. (26) is not empty. We have at least one special and interesting class of Ψ's provided by the Inomata[23] condition $\Psi_{\cdot\mu} = -(1/2)\dot{\beta}H_\mu\Psi$ which satisfies this equation. Moreover, this class also verifies Eq. (27) and then leads to chiral symmetry breaking for all regimes of β, resulting in an explicit solution.

In the case of massive fermions, we expect that each fermion possesses a characteristic energy scale in which this nonlinear coupling is relevant, i.e. $\beta = \beta(m)$ where m is the mass. Thus, this kind of self-interaction does not appear at low energies and the chiral symmetry should be broken by the mass term. On the other hand, the SM takes into account self-interaction only as quantum corrections. Then, it should be carefully examined if quantum corrections could model such generalized NJL self-interaction and if it would be distinguished from our proposal. Nowadays there are only phenomenological properties about the right-handed neutrinos[21] and no observation has been successful. The attempt to introduce specific values to this model is postponed to a more experimental work.

Note that, as a drawback, there is no conservation of the probability current of the spinor field obtained in the Minkowski space through the DB. This can be explained by two different ways: the spinor field perturbations propagate along geodesics in the curved space but not in Minkowski space; this case is then equivalent to the scalar one.[2] The second possible explanation states that the chiral symmetry breaking does also break the unitarity of the Dirac theory. To avoid this difficulty, another proposal is possible by changing the form of the curved geometry. Instead of Eq. (1), if we consider an effective metric of the form

$$\hat{g}^{\mu\nu} = \alpha_1 g^{\mu\nu} + 2\alpha_2 H^\mu H^\nu \,, \tag{31}$$

where $\alpha_1 \neq 1$ and α_2 are initially two arbitrary and independent functions of the scalars A and B. The current is now conserved in both the effective curved and flat spaces provided $\alpha_1 = \alpha_1(\alpha_2)$; that is only one parameter is free. However, preliminary attempts indicate that the dynamics in this case is no longer given by the NJL models. So, this would deviate from our original proposal and should be discussed in a future work.

Acknowledgments

References

1. M. Novello and E. Goulart, *Class. Quantum Grav.* **28**, 145022 (2011).
2. E. Goulart, M. Novello, F. T. Falciano and J. D. Toniato, *Class. Quantum Grav.* **28**, 245008 (2011).

3. M. Novello, F. T. Falciano and E. Goulart, arXiv:1111.2631.
4. M. Novello, E. Bittencourt, U. Mosquella, E. Goulart, J. M. Salim and J. D. Toniato, *J. Cosmol. Astropart. Phys.* **06**, 014 (2013).
5. M. Novello and E. Bittencourt, *Int. J. Mod. Phys. A* **29**, 1450075 (2014).
6. T. Hatsuda and T. Kunihiro, *Phys. Rep.* **247**, 221 (1994).
7. R. P. Feynman, F. B. Morinigoe and W. G. Wagner, *Feynman Lectures on Gravitation* (Addison Wesley, MA, 1995).
8. I. D. Soares, *Proc. II Brazilian School of Cosmology and Gravitation*, ed. M. Novello (J. Sasson & Cia., Rio de Janeiro, 1980).
9. A. Inomata, *Phys. Rev. D* **18**, 3552 (1978).
10. J. D. Bekenstein, *The Sixth Marcel Grossmann Meeting on General Relativity*, ed. H. Sato (World Scientific, Singapore, 1992).
11. J. D. Bekenstein, *Phys. Rev. D* **48**, 3641 (1993).
12. J. Magueijo, *Phys. Rev. D* **79**, 043525 (2009).
13. N. Kaloper, *Phys. Lett. B* **583**, 1 (2004).
14. M. Novello and E. Bittencourt, *Phys. Rev. D* **86**, 124024 (2012).
15. M. Novello and E. Bittencourt, *Gen. Relativ. Gravit.* **45**, 1005 (2013).
16. E. Cartan, *Annu. Sci. Ec. Norm. Sup.* **40**, 325 (1923).
17. H. Weyl, *Z. Phys.* **56**, 330 (1929).
18. M. Novello, *Europhys. Lett.* **80**, 41001 (2007).
19. W. I. Fushchych and R. Z. Zhdanov, *Phys. Rep.* **172**, 123 (1989).
20. Particle Data Group (J. Beringer *et al.*), *Phys. Rev. D* **86**, 010001 (2012).
21. M. Drewes, *Int. J. Mod. Phys. E* **22**, 1330019 (2013).
22. L. Canetti, M. Drewes and M. Shaposhnikov, *Phys. Rev. Lett.* **110**, 061801 (2013).
23. A. Inomata and W. A. McKinley, *Phys. Rev.* **140**, 1467 (1965).